

Spring 2022

INTRODUCTION TO COMPUTER VISION

Atlas Wang

Assistant Professor, The University of Texas at Austin

Visual Informatics Group@UT Austin
<https://vita-group.github.io/>

Famous tale in computer vision

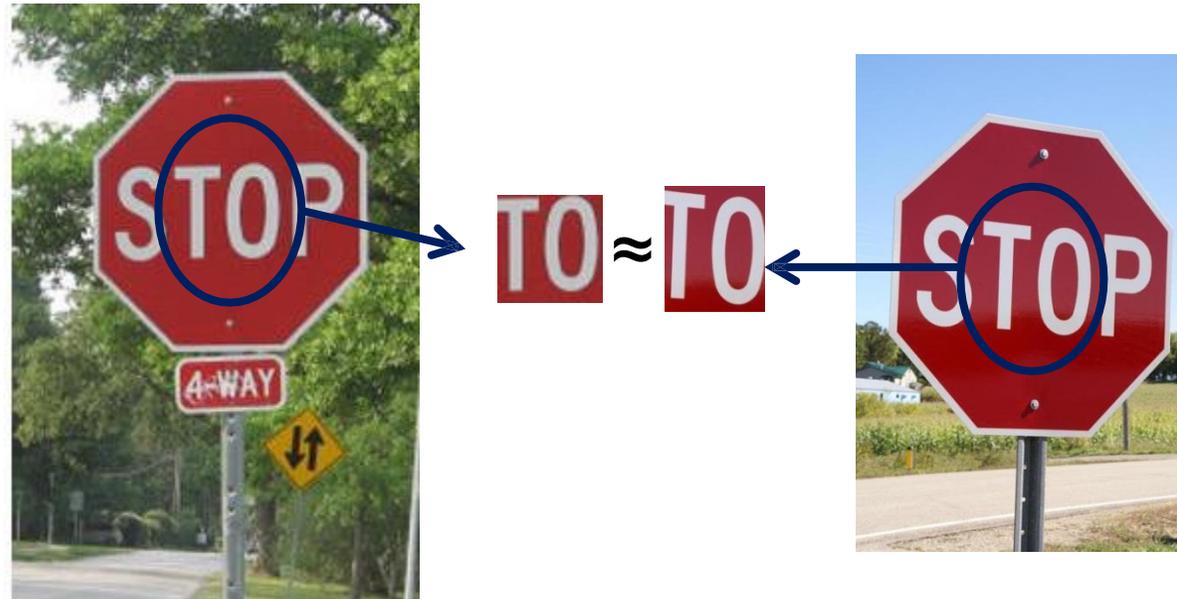
- Once, a CMU graduate student asked the famous computer vision scientist **Takeo Kanade**: *"What are the three most important problems in computer vision?"*
- Takeo replied: **"Correspondence, correspondence, correspondence!"**



Visual Correspondence across views

Matching points, patches, edges, or regions across images.

- *Sparse or local correspondence* (picking some “keypoints”)
- *Dense correspondence* (at every pixel)



Fundamental to Applications

- Image alignment
- 3D reconstruction
- Motion tracking (robots, drones, AR)
- Indexing and database retrieval
- Object recognition



Example application: Panorama stitching

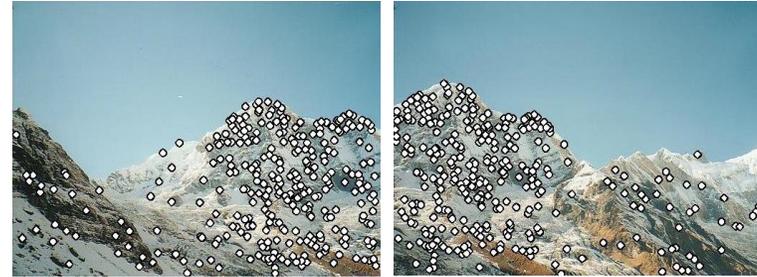
We have two images –
how do we estimate how to overlay them?



Local features: main components

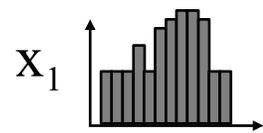
1) Detection:

Find a set of distinctive key points.

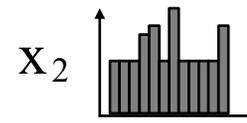
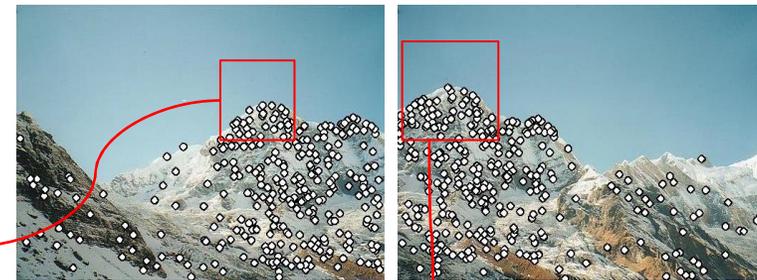


2) Description:

Extract feature descriptor around each interest point as vector.



$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

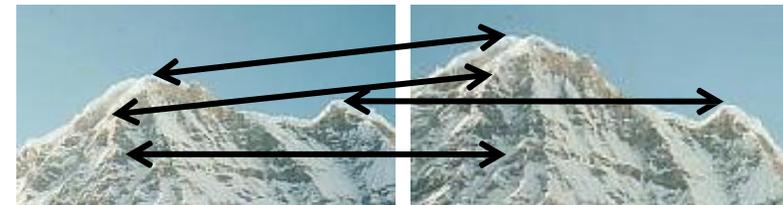


$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) Matching:

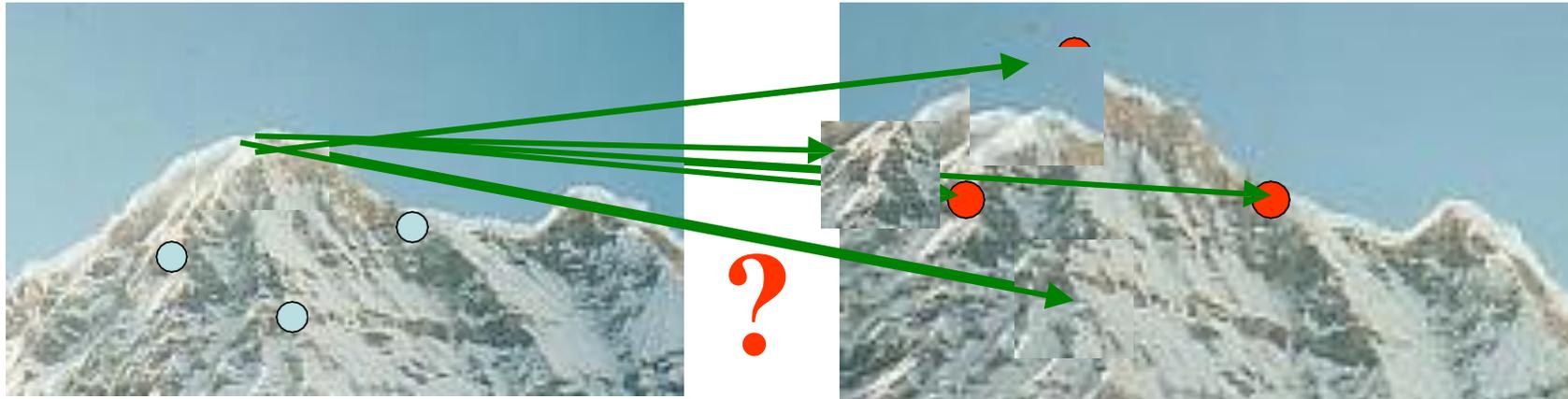
Compute distance between feature vectors to find correspondence.

$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$



Goal: Distinctiveness

We want to be able to reliably determine which point goes with which.



May be difficult in structured environments
with repeated elements

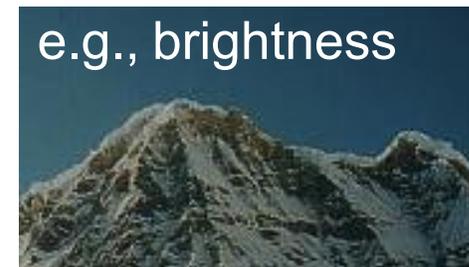
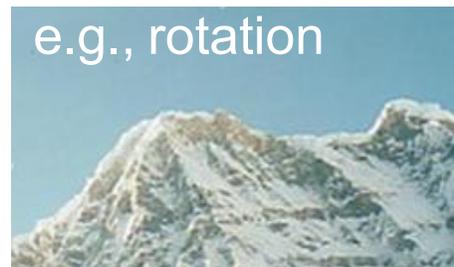
Goal: Repeatability

We want to detect (at least some of) the same points in both images.



With these points, there's no chance to find true matches!

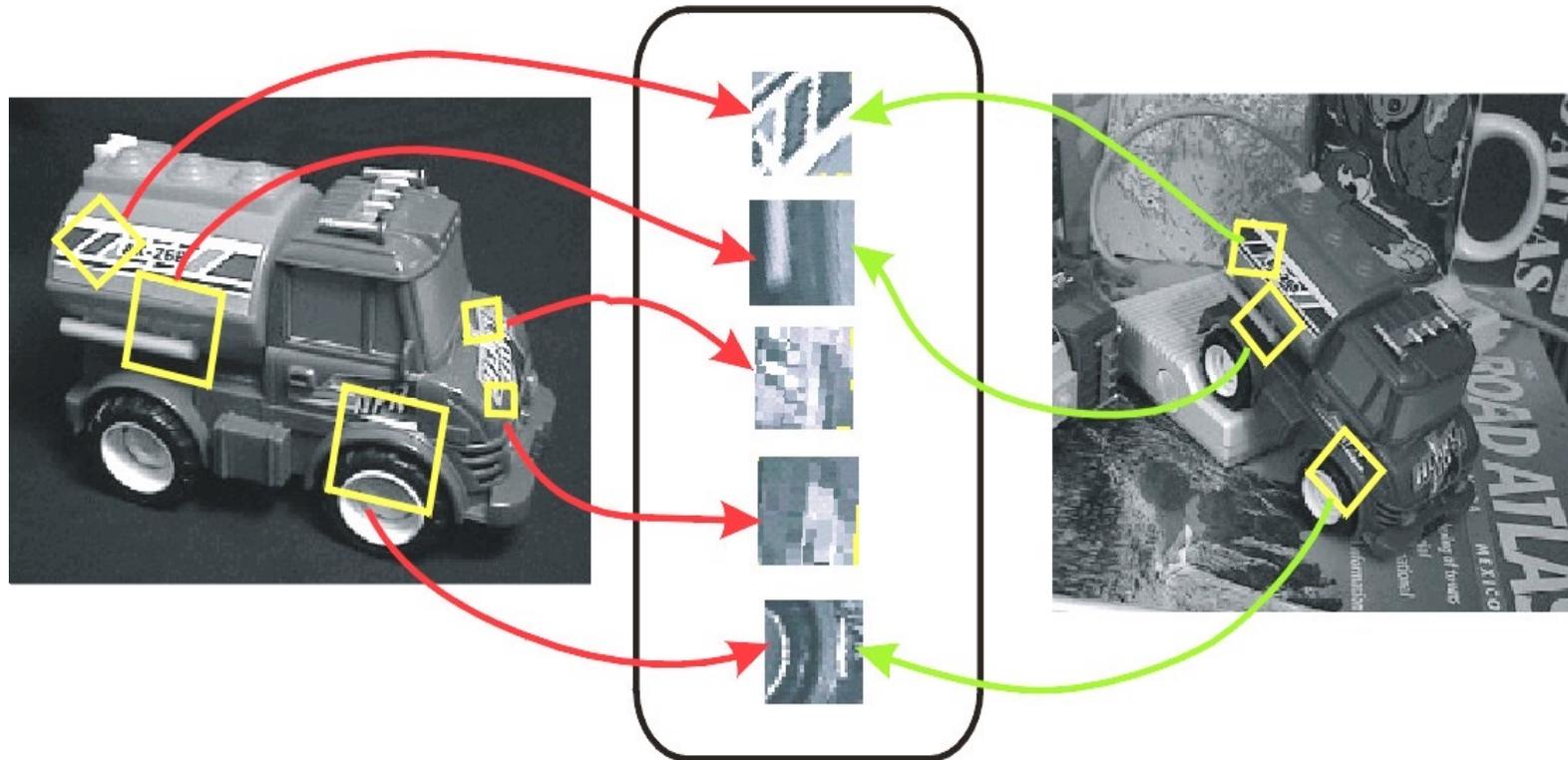
Under geometric and photometric variations.



Example: Object Detection

Finding *distinctive* and *repeatable* feature points can be difficult when we want our features to be invariant to large transformations:

- geometric variation (translation, rotation, scale, perspective)
- appearance variation (reflectance, illumination)



Keypoint Descriptors

Goal: Compactness and Efficiency

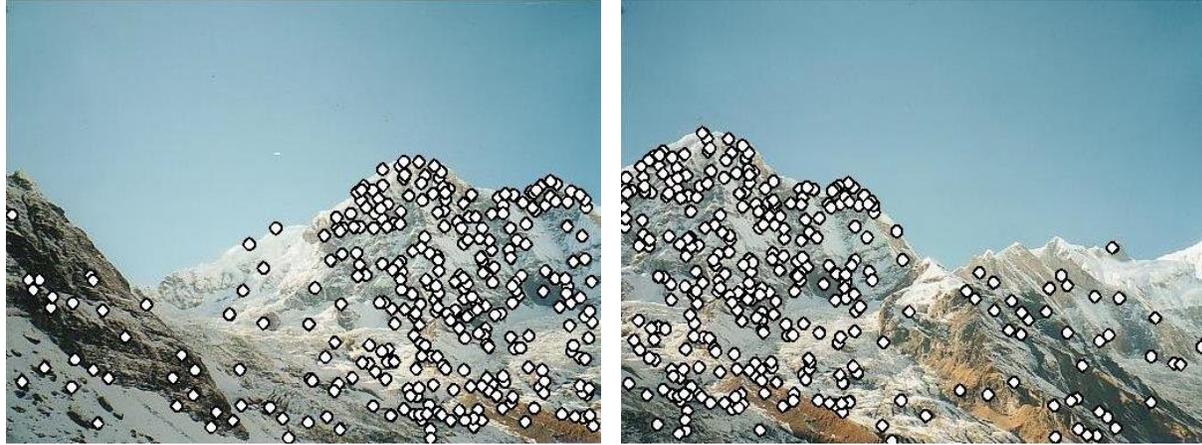
We want the representation to be as small and as fast as possible

- Much smaller than a whole image

Sometimes, we'd like to run the detection procedure *independently* per image

- Match just the compact descriptors for speed.
- *Difficult!* We don't get to see 'the other image' at match time, e.g., object detection.

Characteristics of good features



Distinctiveness

Each feature can be uniquely identified

Repeatability

The same feature can be found in several images despite differences:

- geometrically (translation, rotation, scale, perspective)
- photometrically (reflectance, illumination)

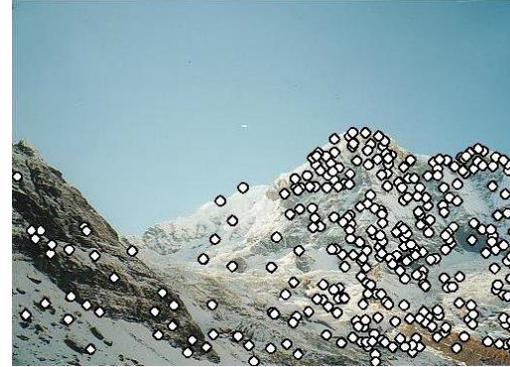
Compactness and efficiency

Many fewer features than image pixels; run independently per image

Local features: main components

1) Detection:

Find a set of distinctive key points.



2) Description:

Extract feature descriptor around each interest point as vector.

3) Matching:

Compute distance between feature vectors to find correspondence.

Detection: Basic Idea

We do not know which other image locations the feature will end up being matched against ...

But can compute how stable a location is in appearance with respect to small variations in its position

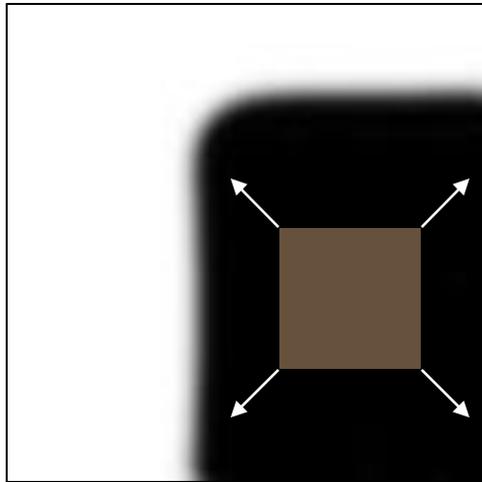
Something that “meaningfully stands out”!

Strategy: Compare image patch against local neighbors

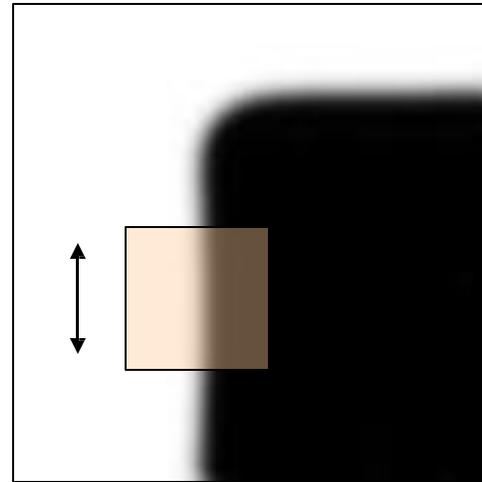
Detection: Basic Idea

Recognize corners by looking at small window.

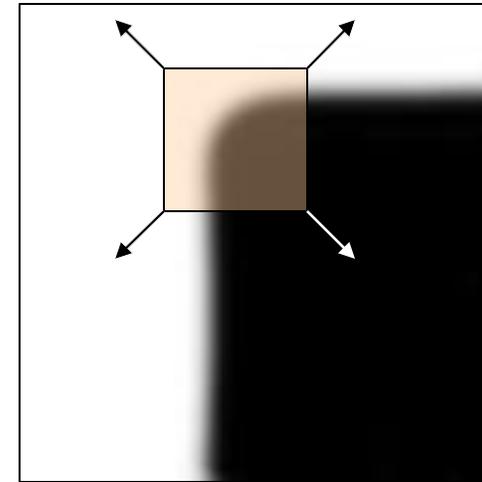
We want a window shift in *any direction* to give *a large change* in intensity.



“Flat” region:
no change in
all directions

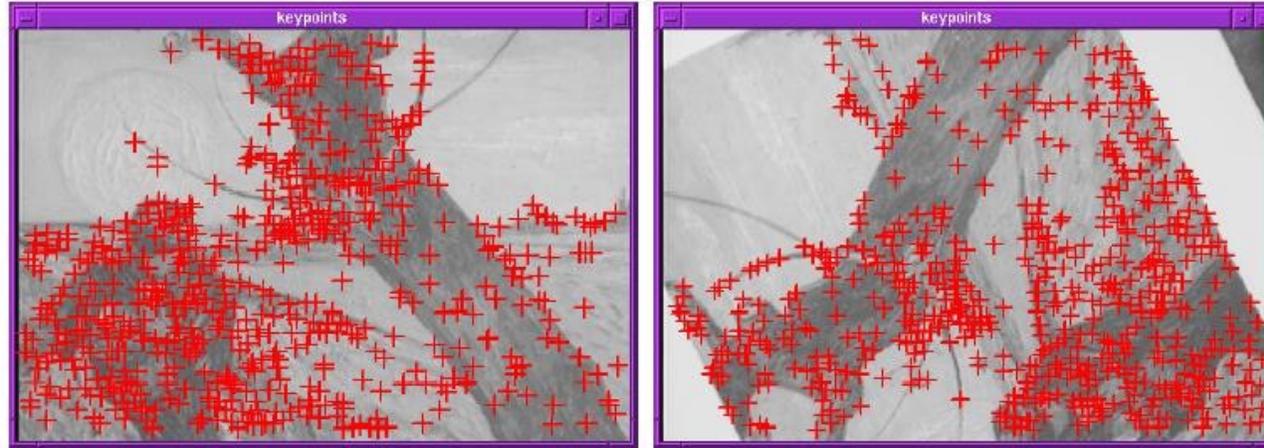


“Edge”:
no change
along the edge
direction



“Corner”:
significant
change in all
directions

Finding Corners



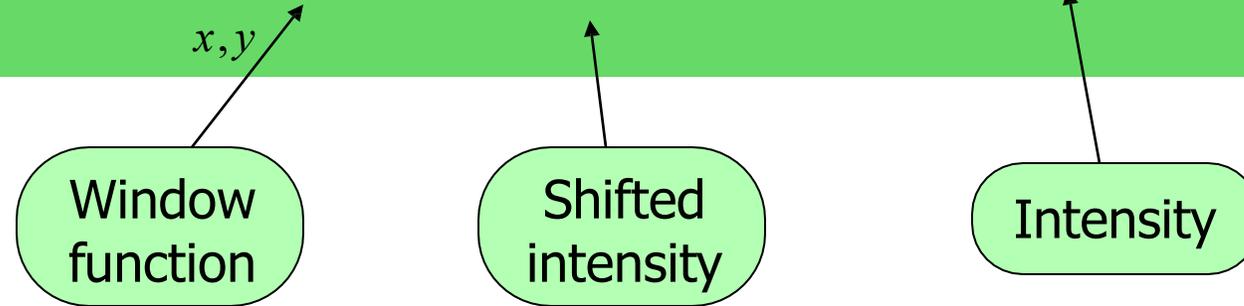
- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)
Proceedings of the 4th Alvey Vision Conference: pages 147--151.

Corner Detection by Auto-correlation

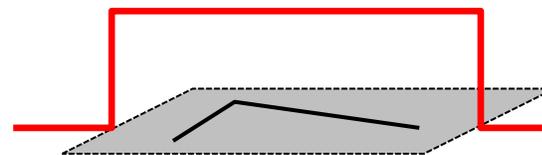
Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u, v) = \sum w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

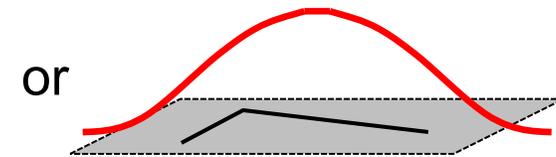


Also called 'sum of squared differences'

Window function $w(x,y) =$



1 in window, 0 outside



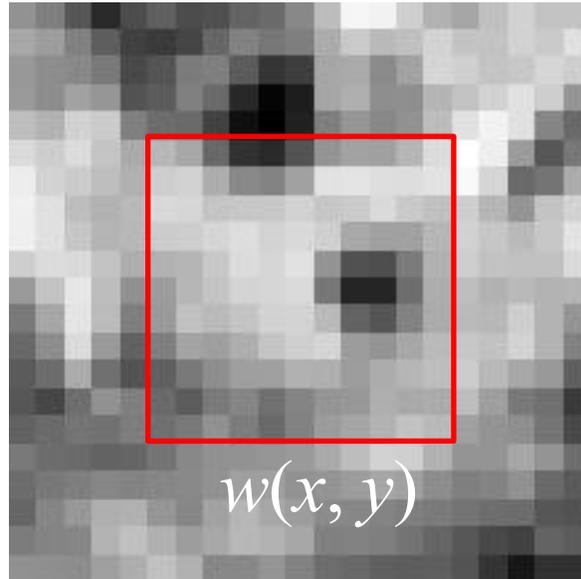
Gaussian

Corner Detection by Auto-correlation

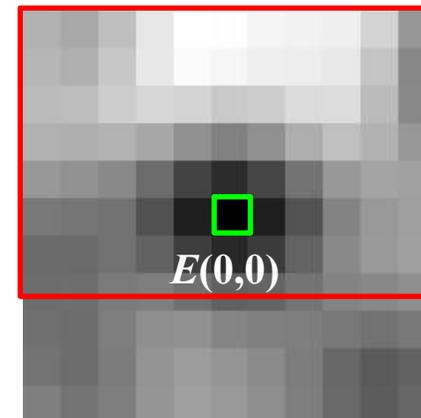
Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$

$I(x, y)$



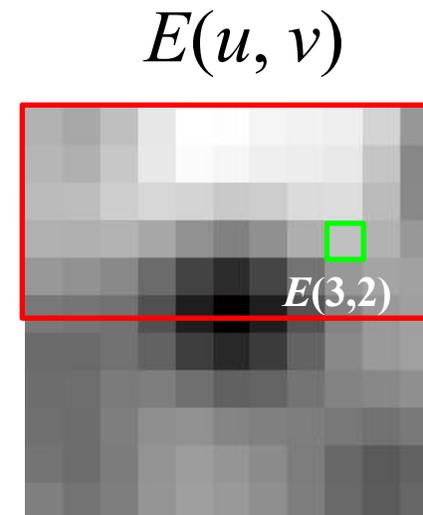
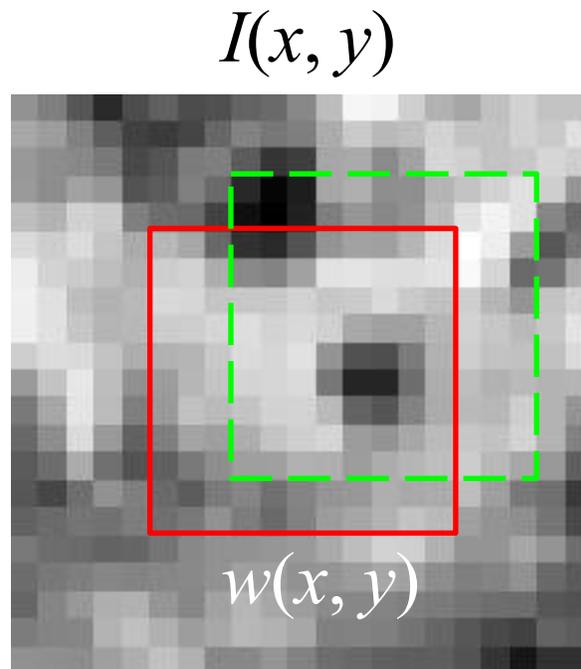
$E(u, v)$



Corner Detection by Auto-correlation

Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2$$



Corner Detection by Auto-correlation

Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]$$

**We want to discover how E behaves for small shifts
(corner = function value change fast w.r.t small shifts)**

But this is very slow to compute naively.

$O(\text{window_width}^2 * \text{shift_range}^2 * \text{image_width}^2)$

$O(11^2 * 11^2 * 600^2) = 5.2$ billion of these 14.6k
ops per image pixel



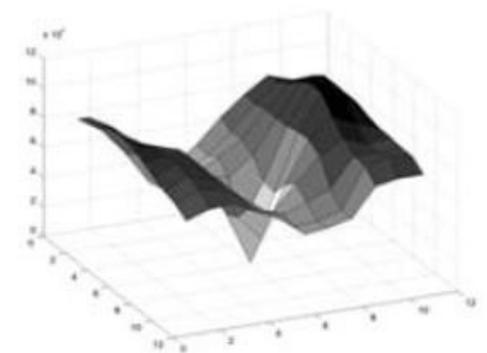
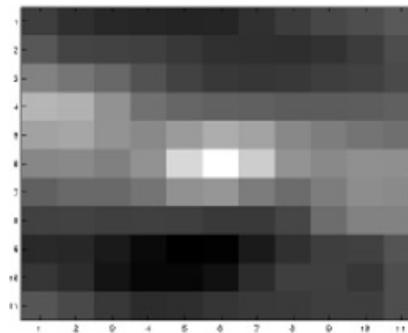
Corner Detection by Auto-correlation

Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]$$

....But we know the response in E that we are looking for – **strong peak!**

- *E needs to “change” fast w.r.t. u & v*
- *(from $u = 0, v = 0$)*



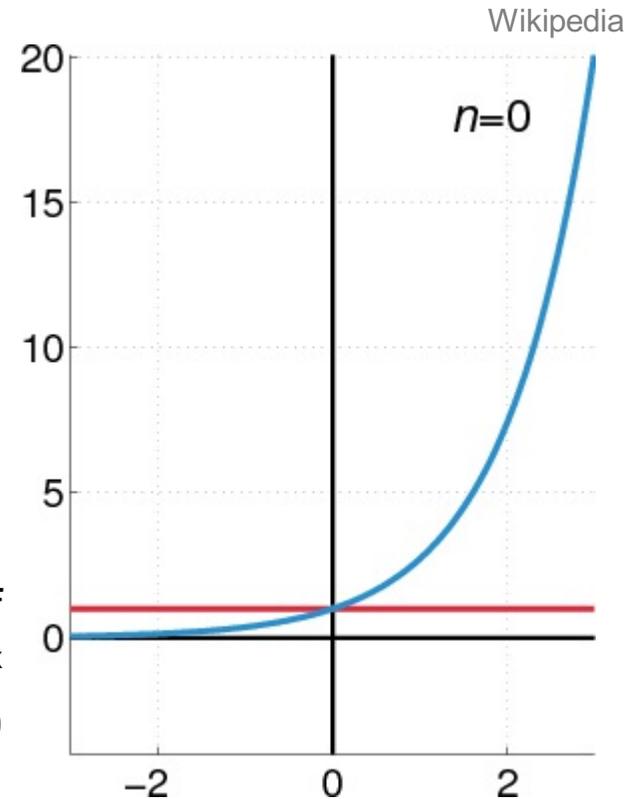
Recall: Taylor series expansion

A function f can be represented by an infinite series of its derivatives at a single point a :

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

As we care about window centered, we set $a = 0$ (MacLaurin series)

Approximation of
 $f(x) = e^x$
centered at $f(0)$



Corner Detection: Mathematics (Simplified)

- First-order Taylor approximation for small shifts (u, v) :

$$I(x + u, y + v) \approx I(x, y) + I_x u + I_y v$$

(Why first-order is good enough?)

- Let's plug this into $E(u, v)$:

$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

(We ignore W here for simplicity)

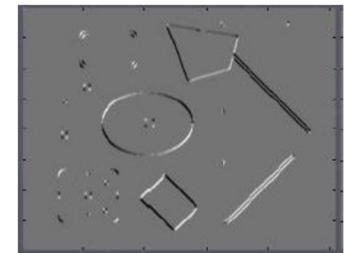
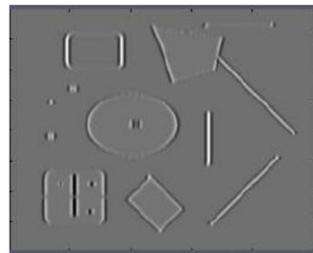
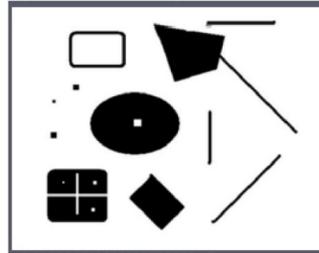
$$\approx \sum_{(x,y) \in W} [I(x, y) + I_x u + I_y v - I(x, y)]^2$$

$$= \sum_{(x,y) \in W} [I_x u + I_y v]^2 = \sum_{(x,y) \in W} I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2$$

Corners as distinctive interest points

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad M = \sum w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives
(averaged in neighborhood of a point)



Notation:

$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$

$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$

$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$

Corners as distinctive interest points

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad M = \sum w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives
(averaged in neighborhood of a point)

Reminder/Refresher:

- Our goal is to find (x, y) likely at corner. (u, v) denotes a small neighborhood near (x, y)
- $E(u, v)$ is evaluated at each (x, y) . Its “parameter” depends on (x, y) , e.g., M
- For each (x, y) , we want to find “extreme” values for $E(u, v)$ -- *now reducing to analyzing M*
- M encodes the “**variation**” level of $E(u, v)$ in the small (u, v) neighborhood – how to decode?

Let's go back to our goal: corner detection

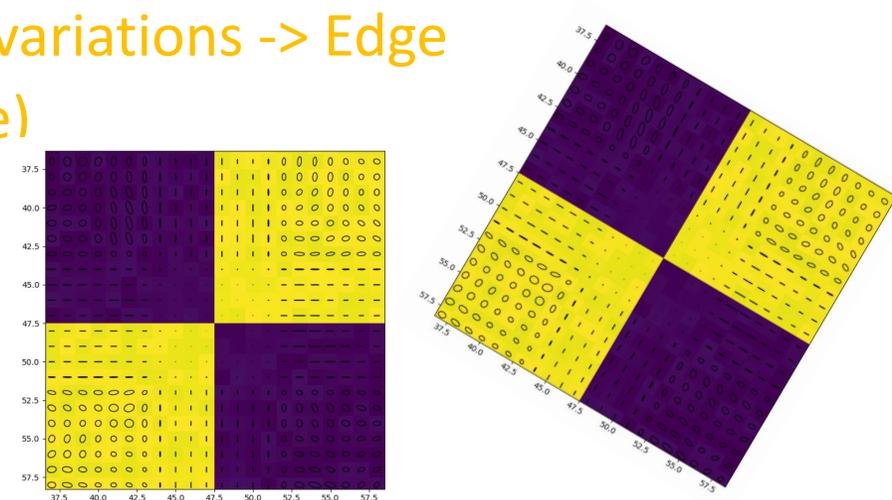
- For detecting “cornerness”:

- Do we care about the change orientation? **No**
- Do we care about the change “steepness”? **Yes, that is “all we need”**

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- So, looking at the M approximation now, what we really want?

- What if $I_x^2, I_y^2, I_x I_y$ are all small? **No variations -> flat area**
- What if only I_x^2 is large? **Only x-direction has large variations -> Edge**
- How about only large I_y^2 , or $I_x I_y$? **Same thing (edge)**
- Then, how about letting $I_x^2, I_y^2, I_x I_y$ all be large?
 - Sufficient, but not necessary...
 - **The missing key: Rotation Invariance**



Eigenvalue Analysis (your old friend: PCA)

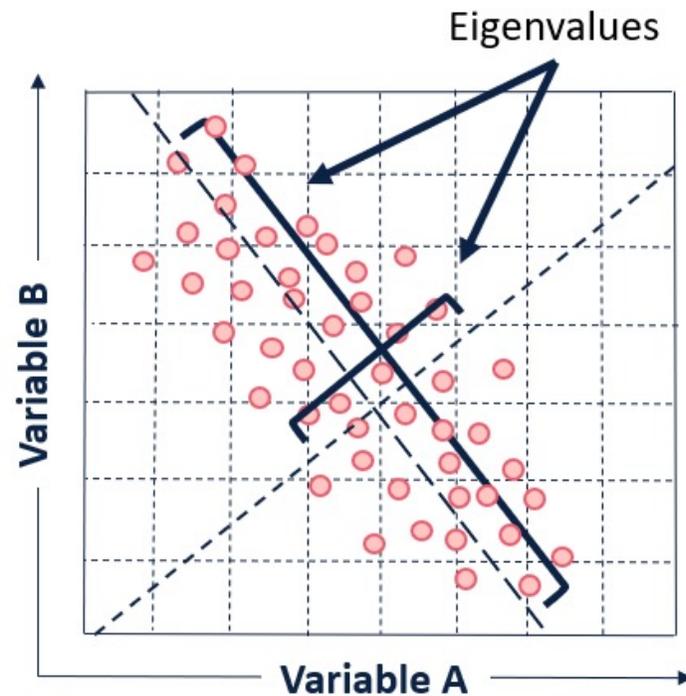
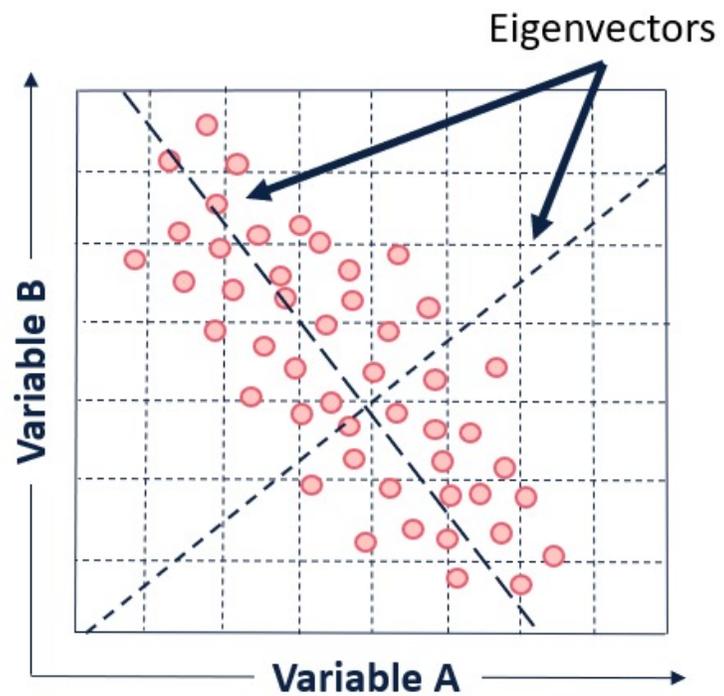
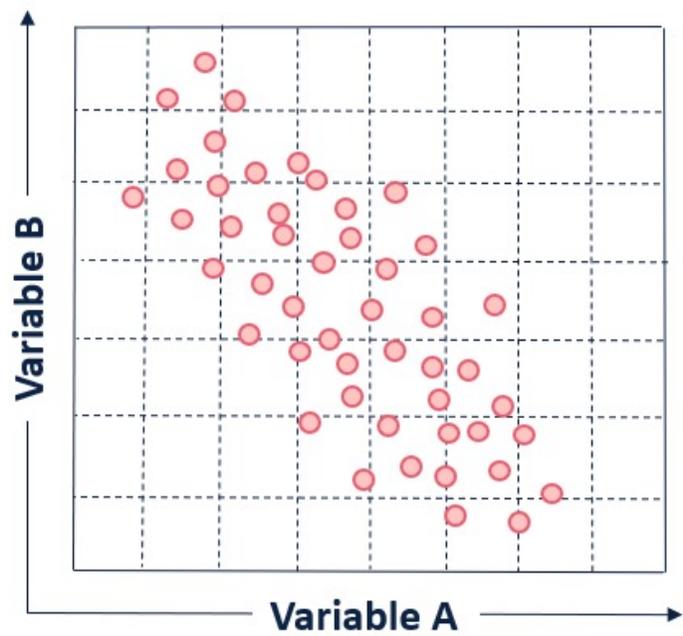
- **Goal:** Describe the “overall intensity variations” in the window, *regardless of rotation!*

... by **eigenvalue analysis**

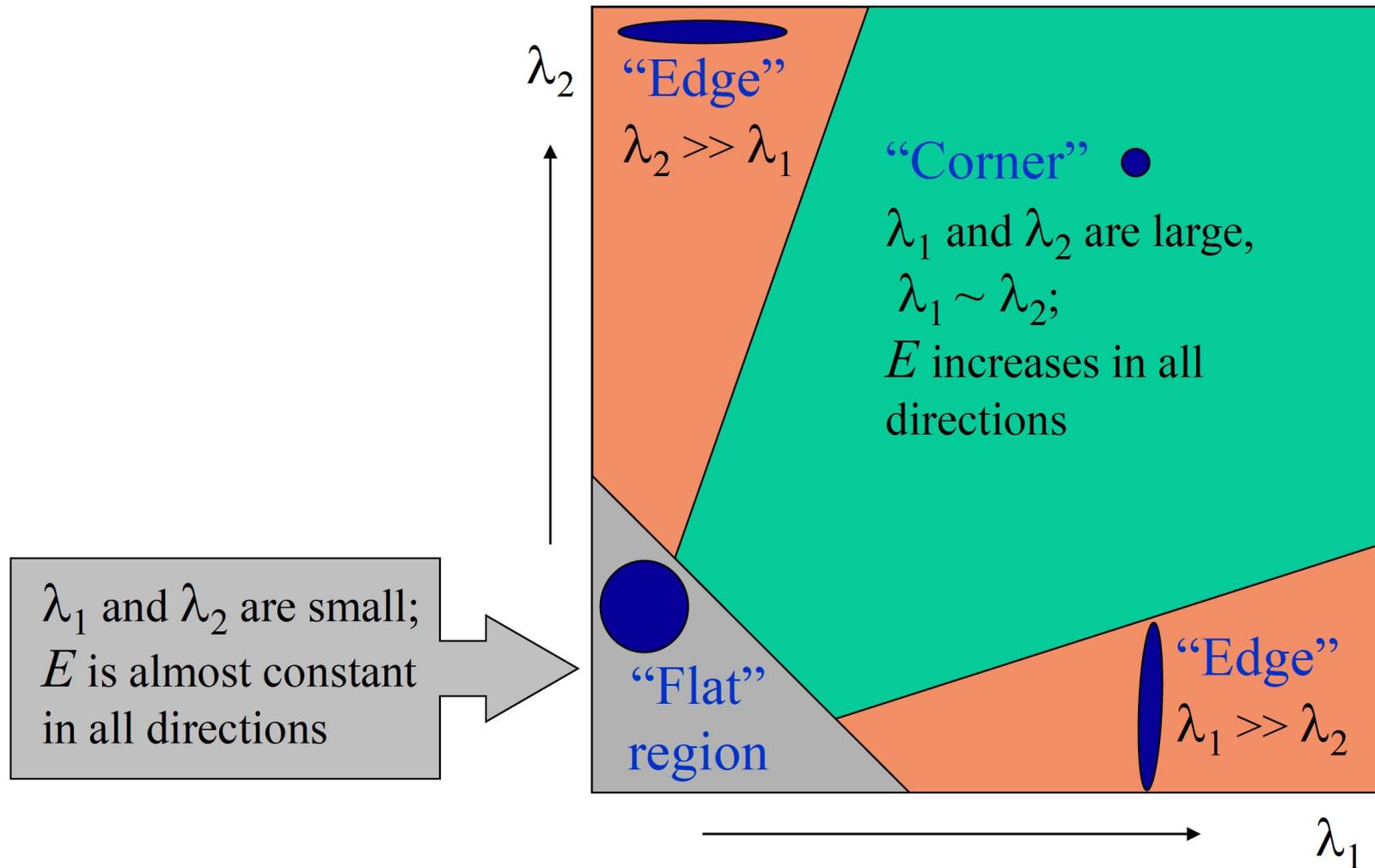
$$E(u, v) \cong [u, v] M \begin{bmatrix} u \\ v \end{bmatrix} \quad \lambda_1, \lambda_2 - \text{eigenvalues of } M$$

What PCA can tell us about the overall “variations”

- *Eigenvectors told us the 1st/2nd/3rd ... major directions of change*
- *Correspondingly, eigenvalues capture “change rate” along each of those directions*



Categorizing image points using M eigenvalues



Categorizing image points using M eigenvalues

Cornerness score:

$$C = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α : some small constant (~ 0.04 to 0.06)

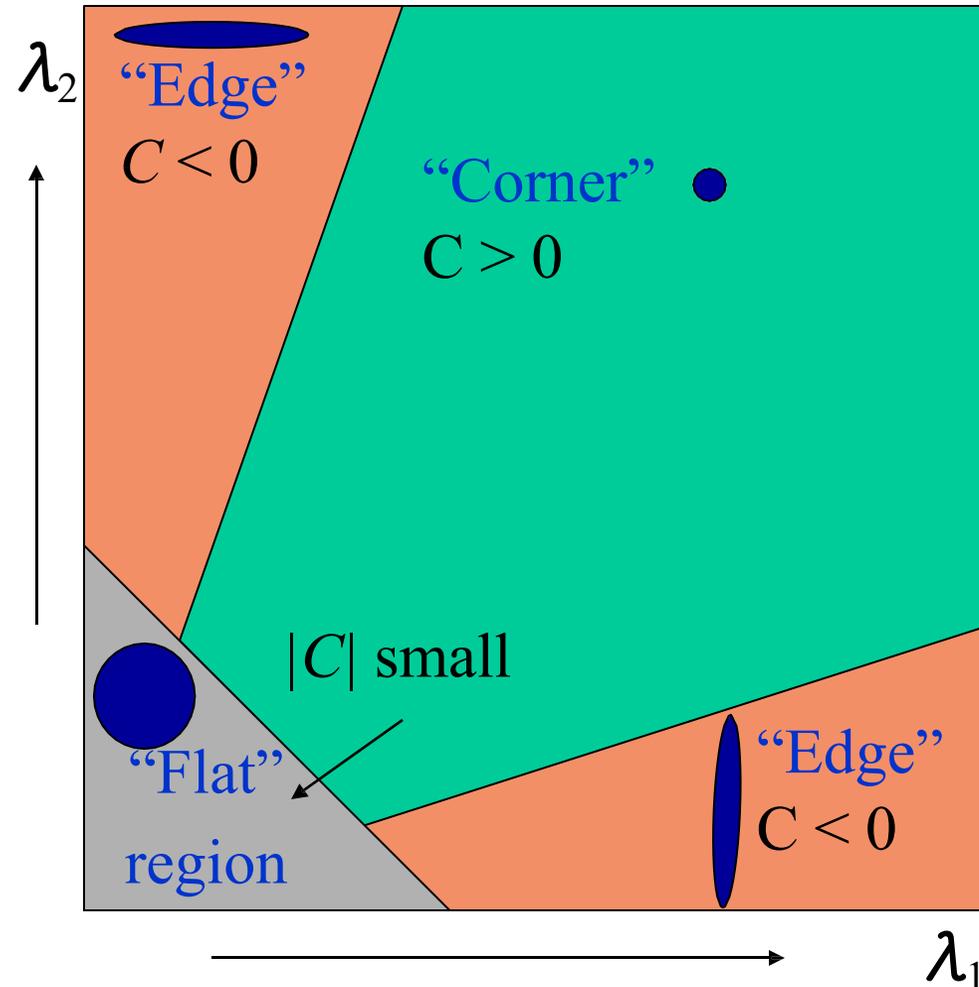
To see why:

Assume $\lambda_1 = k\lambda_2$,

$$C = [k^2 - \alpha(k+1)^2] \lambda_2^2$$

Then analyze: $k^2 - \alpha(k+1)^2$

What if k is very large? very small? around 1?



Categorizing image points using M eigenvalues

Cornerness score:

$$C = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

α : some constant (~ 0.04 to 0.06)

Remember your linear algebra:

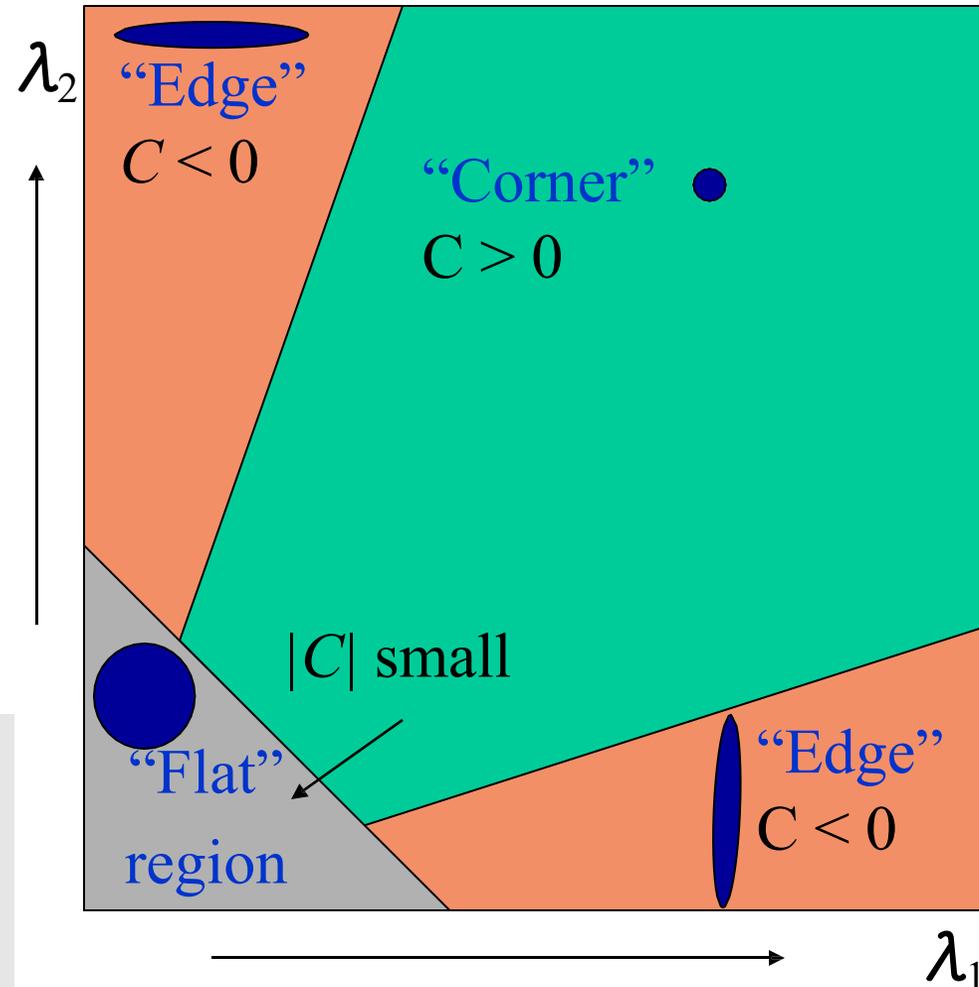
Determinant: $\det(A) = \prod_{i=1}^n \lambda_i = \lambda_1 \lambda_2 \cdots \lambda_n$.
(diagonal matrices)

Trace: $\text{tr}(A) = \sum_i \lambda_i$.

$$C = \det(M) - \alpha \text{Tr}^2(M)$$

Avoids explicit eigenvalue computation!

(many fast algorithms to directly estimate det/Tr)



This is the "notorious" Harris corner detector!

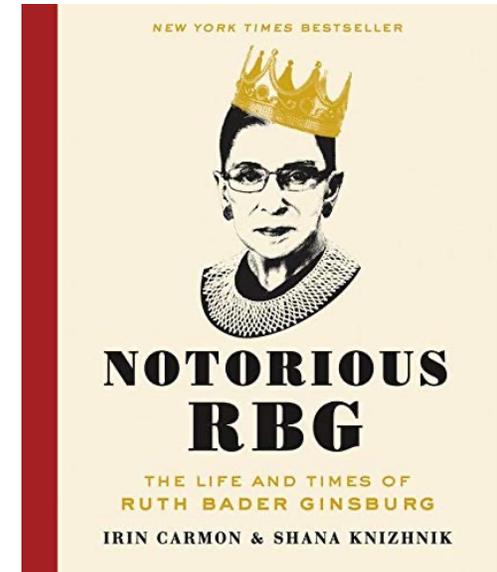
- 1) Compute M matrix for each window to recover the *cornerness* score C .

Note: We can find M purely from the per-pixel image derivatives!

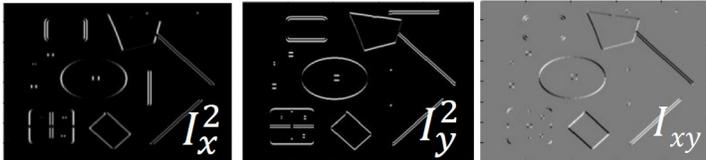
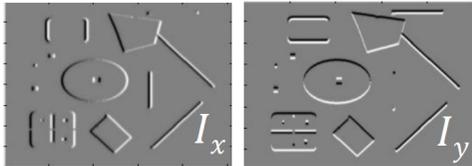
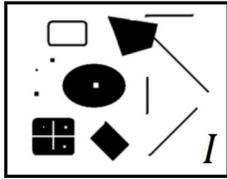
- 2) Threshold to find pixels which give large corner response ($C > \text{threshold}$).

- 3) Find the local maxima pixels, i.e., non-maximal suppression.

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.



Harris Corner Detector [Harris88]



0. Input image

We want to compute M at each pixel.

1. Compute image derivatives (optionally, blur first).

2. Compute M components as squares of derivatives.

3. Gaussian filter $g()$ with width σ

$$= g(I_x^2), g(I_y^2), g(I_x \circ I_y)$$

Reminder: $a \circ b$ is Hadamard product (element-wise multiplication)

4. Compute cornerness

$$\begin{aligned} C &= \det(M) - \alpha \text{trace}(M)^2 \\ &= g(I_x^2) \circ g(I_y^2) - g(I_x \circ I_y)^2 \\ &\quad - \alpha [g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Threshold on C to pick high cornerness

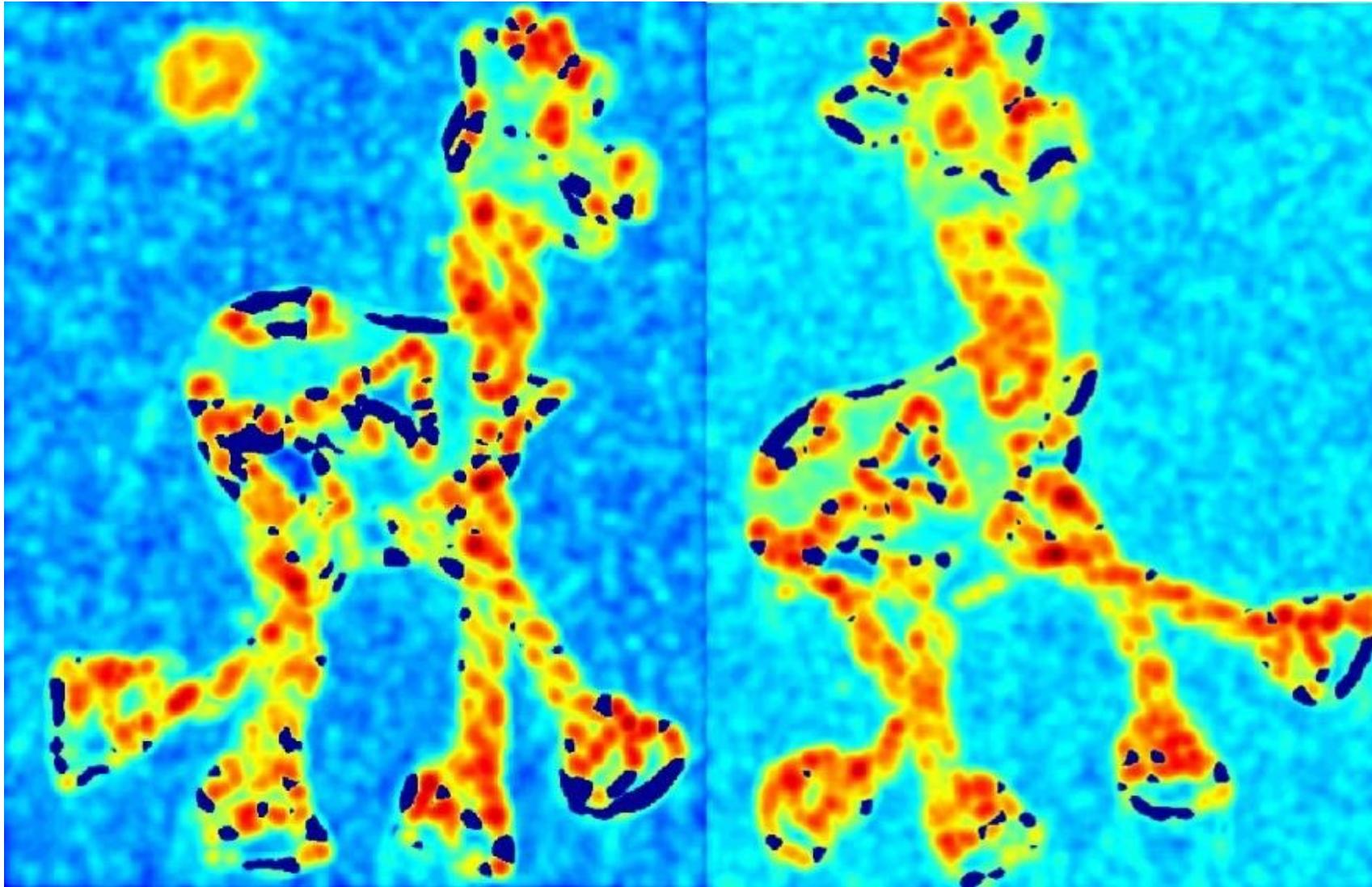
6. Non-maximal suppression to pick peaks.

Harris Detector: Steps



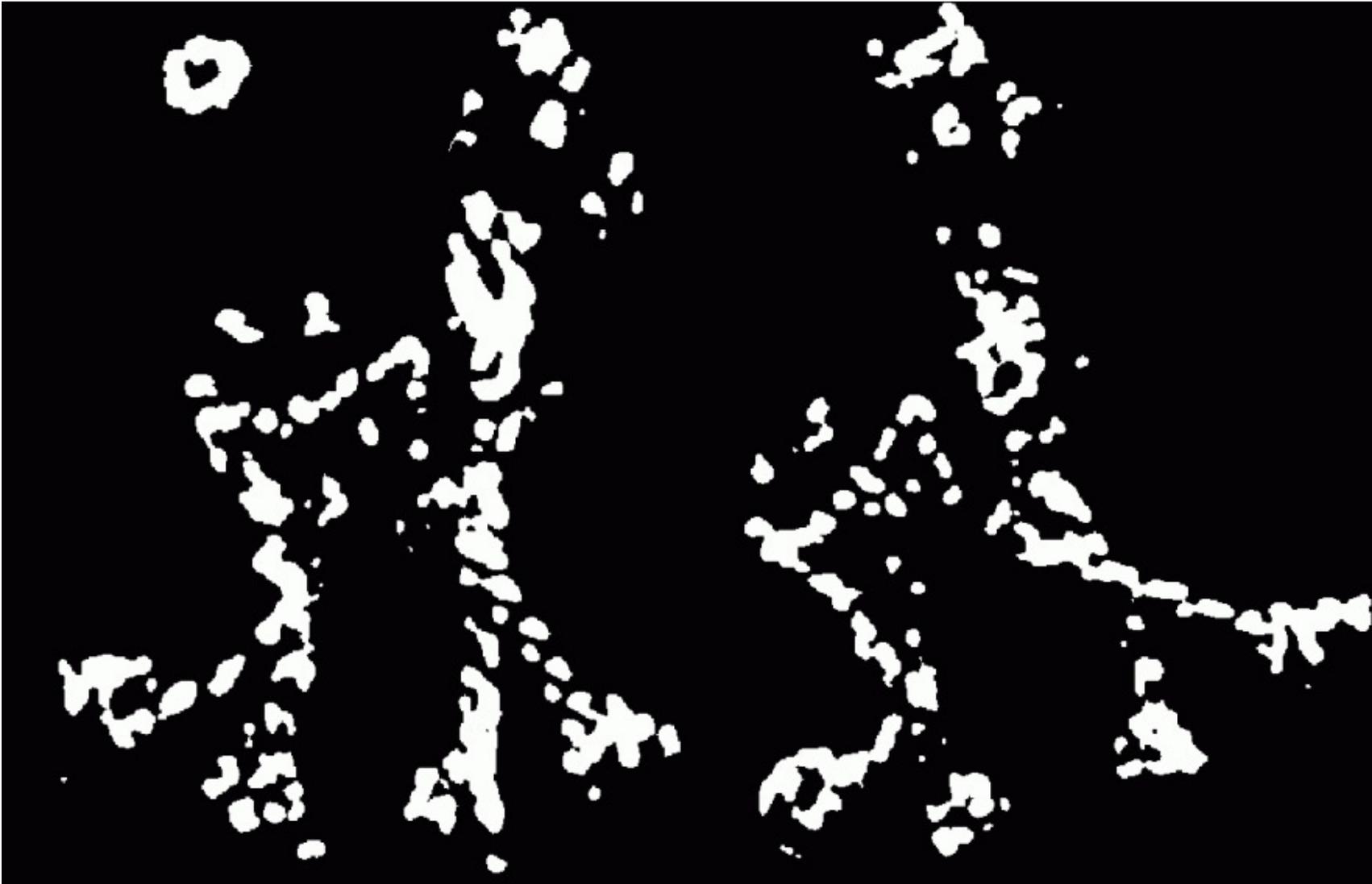
Harris Detector: Steps

Compute corner response C



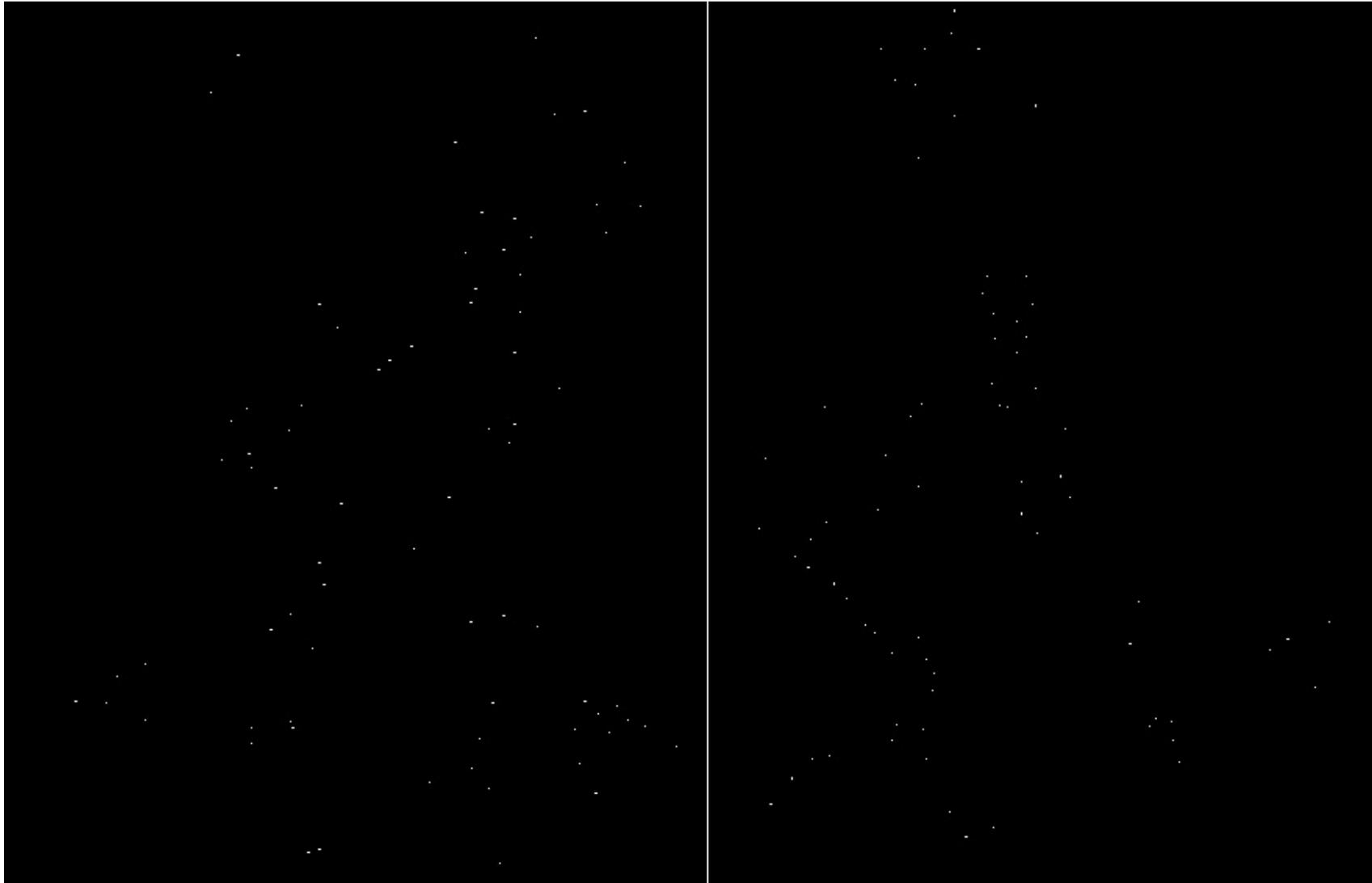
Harris Detector: Steps

Find points with large corner response: $C > \text{threshold}$



Harris Detector: Steps

Take only the points of local maxima of C

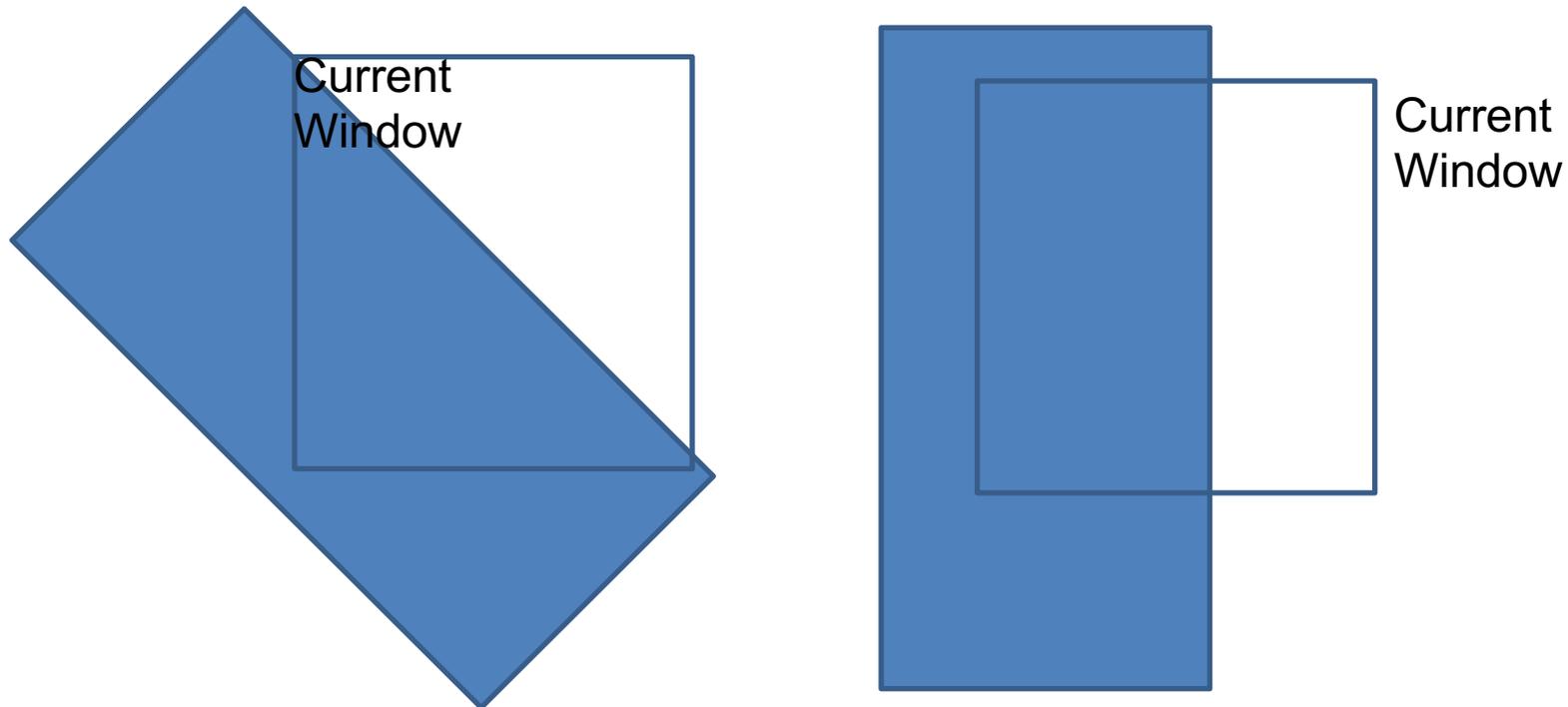


Harris Detector: Steps



Harris Corners – Why so complicated?

- Can't we just check for regions with lots of gradients in the x and y directions (or any specific)?
 - No! A diagonal line or alike would satisfy that criteria



Invariance and covariance

Are locations *invariant* to photometric transformations and *covariant* to geometric transformations?

- **Invariance:** image is transformed and corner locations do not change
- **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

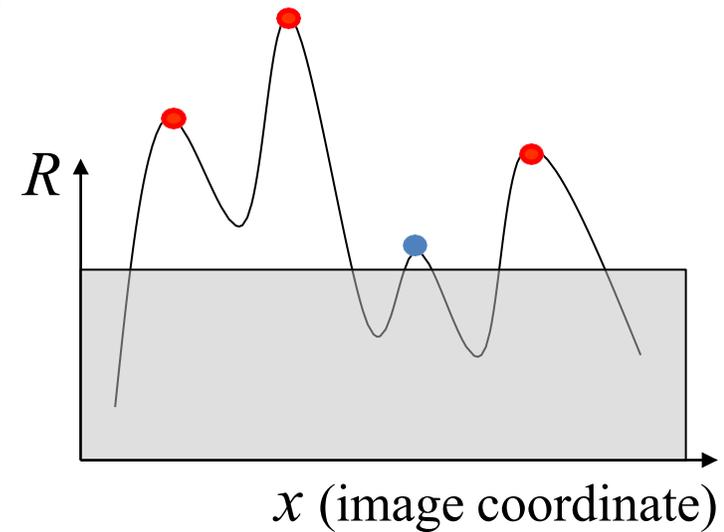
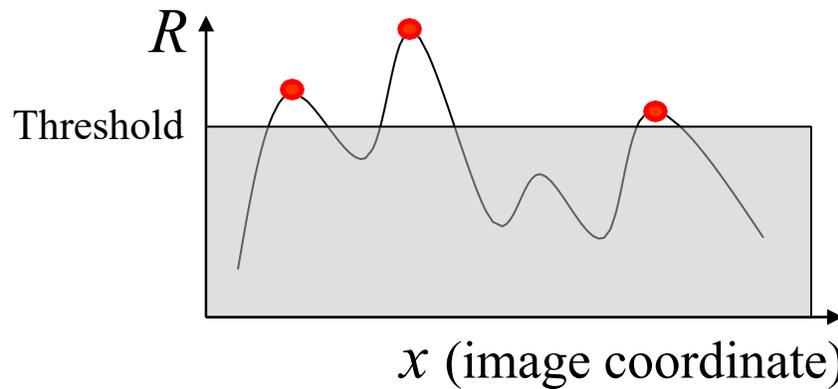


Affine intensity change



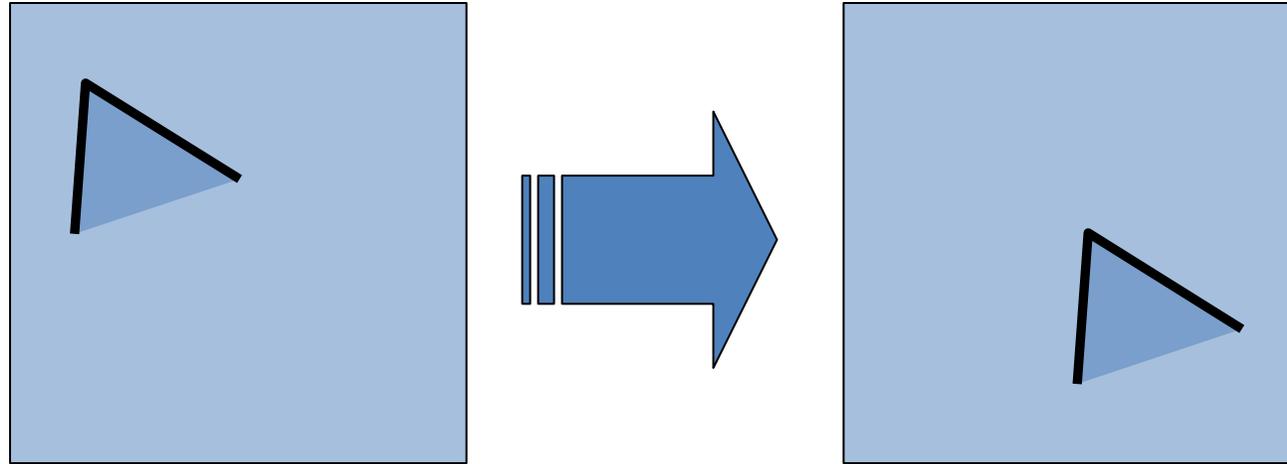
$$I \rightarrow a I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a I$



Partially invariant to affine intensity change

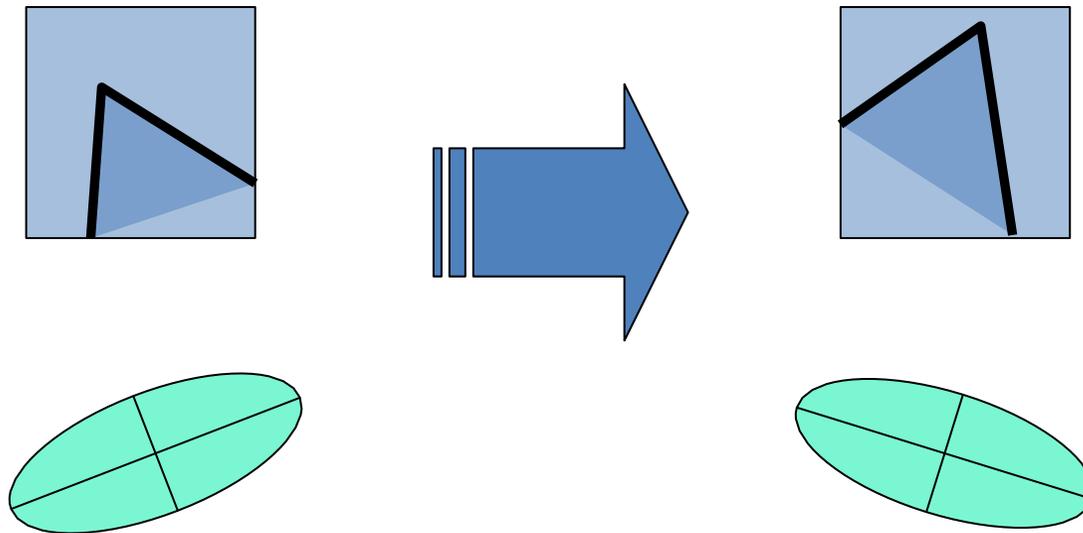
Image translation



- Derivatives and window function are shift-invariant.

Corner location is covariant w.r.t. translation

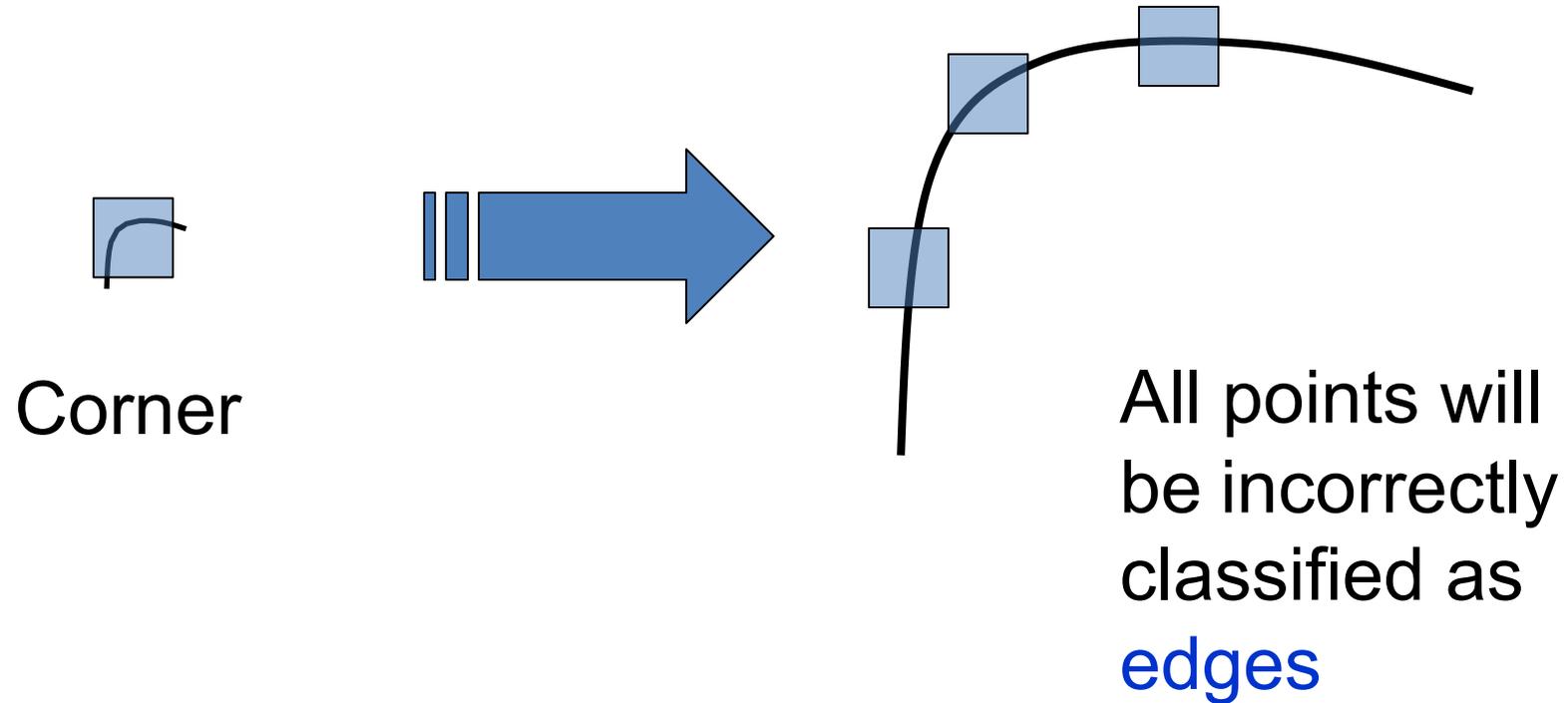
Image rotation



Second moment ellipse rotates but its shape (i.e., eigenvalues) remains the same.

Corner location is covariant w.r.t. rotation

Scaling

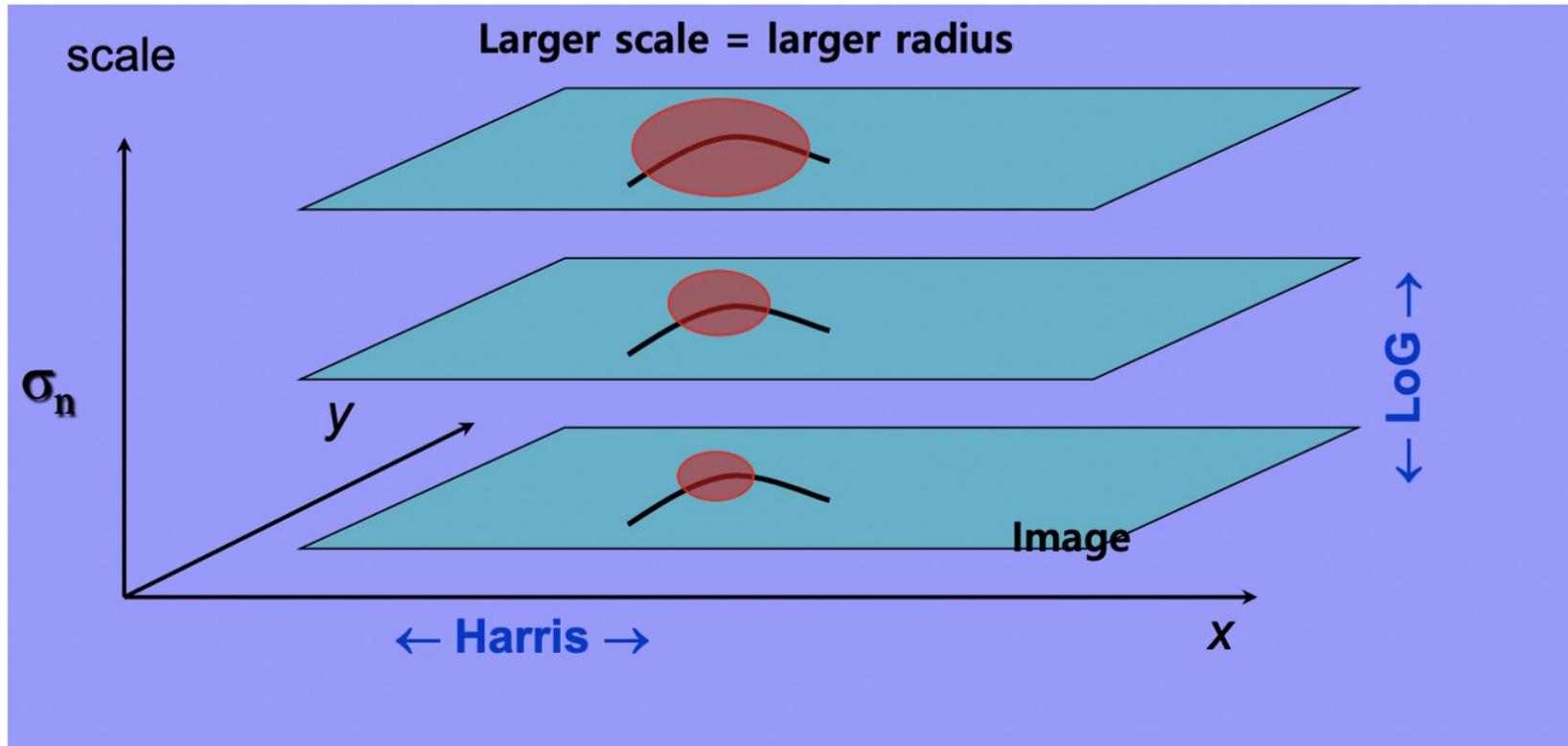


Corner location is not covariant to scaling!

Fixing the scale ambiguity: “Scale Space”

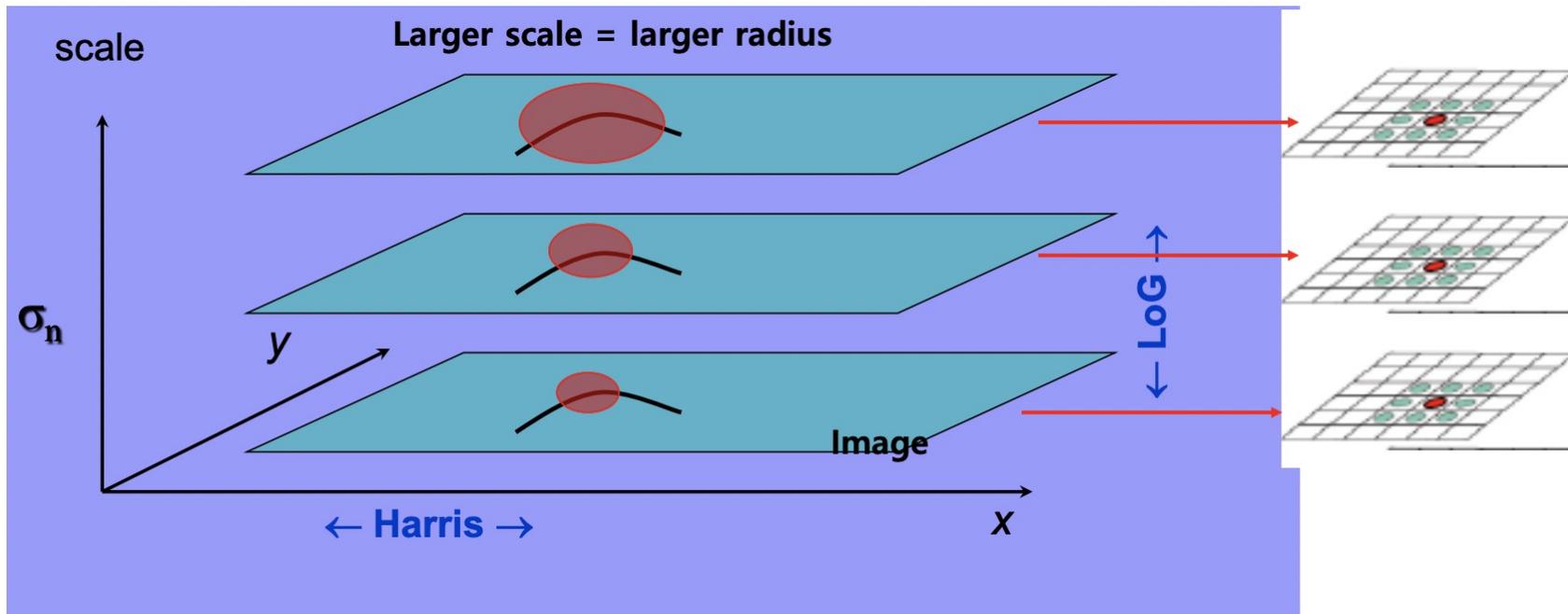
- Given a keypoint in each image of an image pair, we want to determine whether their surrounding image neighborhoods contain the same structure up to an unknown scale factor
- We evaluate a **scale-dependent signature function** on the keypoint neighborhood and plot the resulting value as *a function of the scale*.
 - If the two keypoints correspond to the same structure, then their signature functions will take similar “overall shapes”
 - ... and their corresponding “just right” neighborhood sizes can be determined by searching for scale-space extrema of the signature function.
- Effectively, this procedure builds up a **scale space**

Harris-Laplace detector [Mikolajczyk '01]



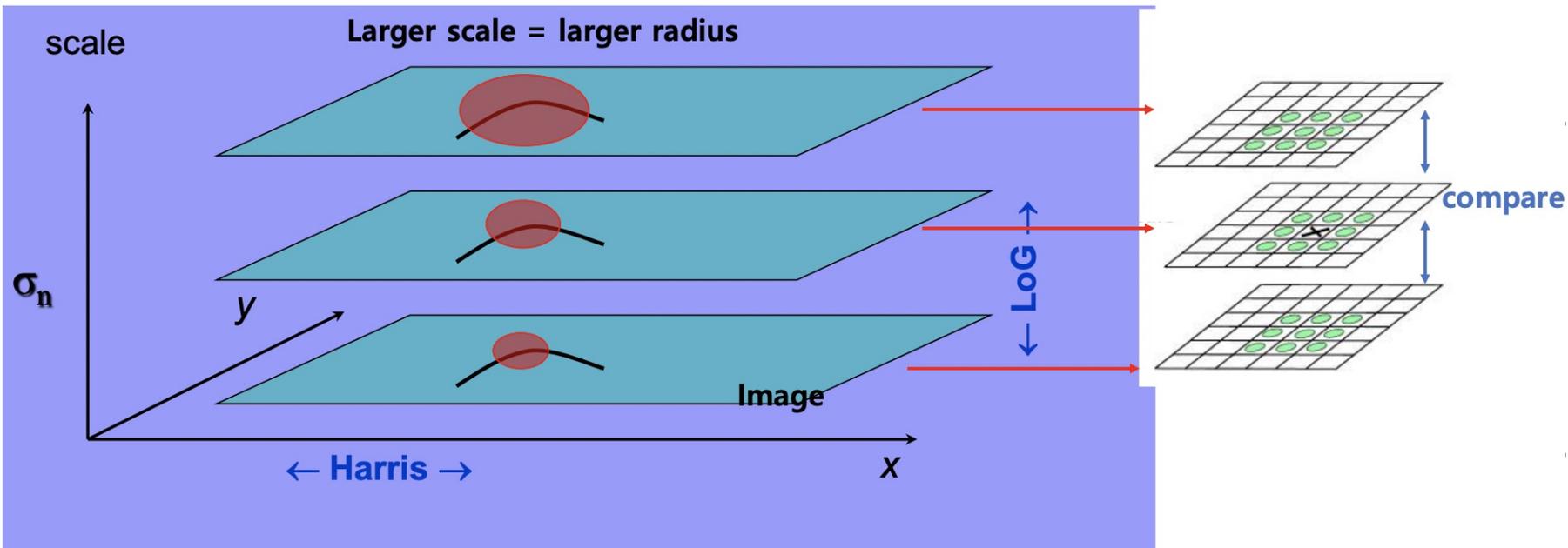
- **Step 1.** Build the Laplacian Pyramid of one image

Harris-Laplace detector [Mikolajczyk '01]



- **Step 1.** Build the Laplacian Pyramid of one image
- **Step 2.** Run the *Harris* detector to compute interest points *at each scale*

Harris-Laplace detector [Mikolajczyk '01]



- **Step 1.** Build the Laplacian Pyramid of one image

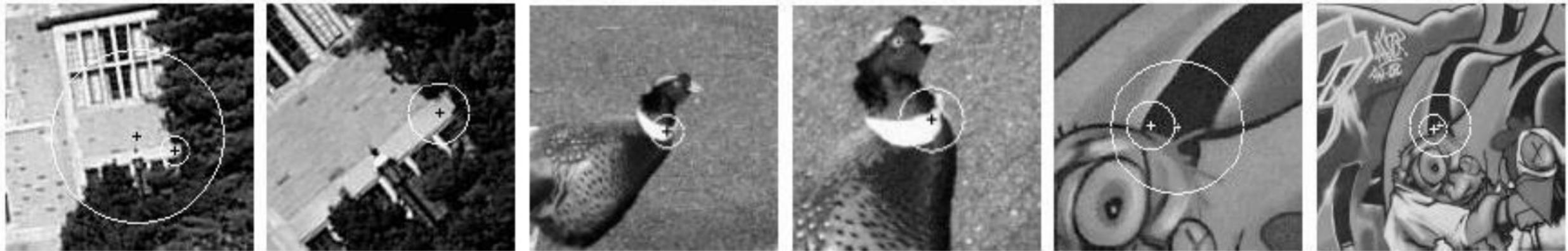
Step 2. Run the *Harris* detector to compute interest points *at each scale*

Step 3. Non-maximal suppression, not only at each scale, **but also at adjacent scales**

Harris-Laplace detector [Mikolajczyk '01]

- **A scale-invariant detector!**

- *Automatically search for the right scale to detect corners, by “multi-scaling then max-pooling”*



Harris-Laplace points

A Longer List of Local Keypoint Detectors...

Table 7.1 Overview of feature detectors.

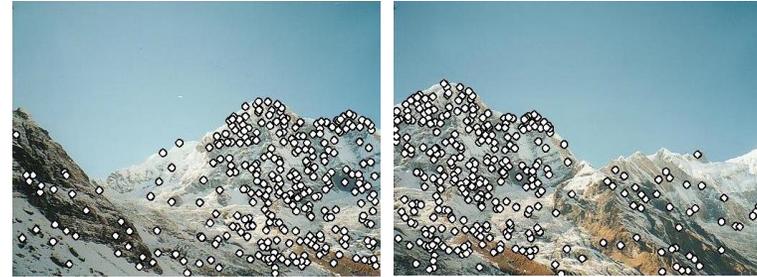
Feature Detector	Corner	Blob	Region	Rotation invariant	Scale invariant	Affine invariant	Localization			
							Repeatability	accuracy	Robustness	Efficiency
Harris	✓			✓			+++	+++	+++	++
Hessian		✓		✓			++	++	++	+
SUSAN	✓			✓			++	++	++	+++
Harris-Laplace	✓	(✓)		✓	✓		+++	+++	++	+
Hessian-Laplace	(✓)	✓		✓	✓		+++	+++	+++	+
DoG	(✓)	✓		✓	✓		++	++	++	++
SURF	(✓)	✓		✓	✓		++	++	++	+++
Harris-Affine	✓	(✓)		✓	✓	✓	+++	+++	++	++
Hessian-Affine	(✓)	✓		✓	✓	✓	+++	+++	+++	++
Salient Regions	(✓)	✓		✓	✓	(✓)	+	+	++	+
Edge-based	✓			✓	✓	✓	+++	+++	+	+
MSER			✓	✓	✓	✓	+++	+++	++	+++
Intensity-based			✓	✓	✓	✓	++	++	++	++
Superpixels			✓	✓	(✓)	(✓)	+	+	+	+

- It is always of interest to collect more points with more detectors, for more possible matches
- Need consider location preciseness, variation robustness, and flexibility in region shapes
- Best choice often application dependent
 - Harris/Harris-Laplace work well for many natural image categories
 - MSER works well for buildings and printed things
 - Although no “silver bullet”, all detectors/descriptors shown here work well in general

Local features: main components

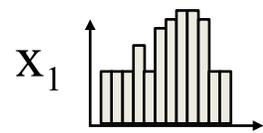
1) Detection:

Find a set of distinctive key points.

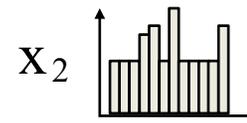
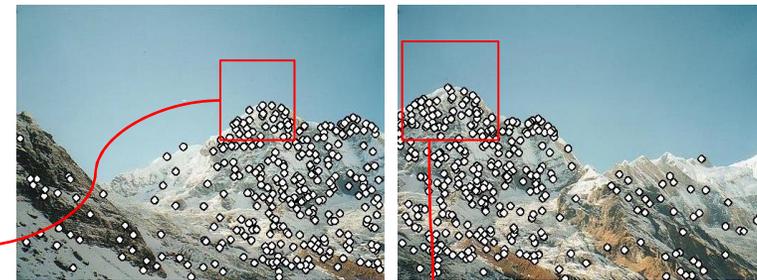


2) Description:

Extract feature descriptor around each interest point as vector.



$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) Matching:

Compute distance between feature vectors to find correspondence.

$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$

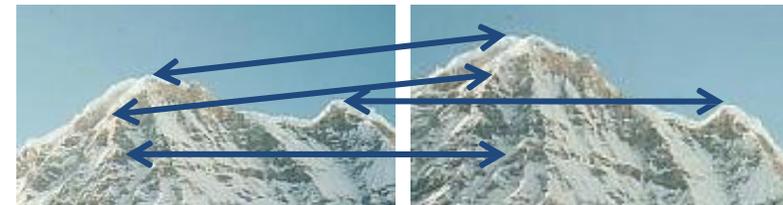


Image patch

Just use the pixel values of the patch



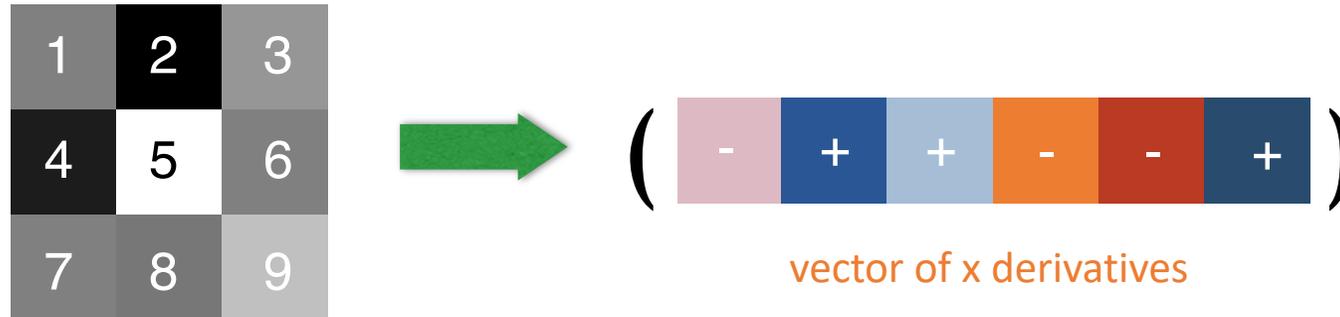
Perfectly fine if geometry and appearance is unchanged (a.k.a. template matching)

What are the problems?

How can you be less sensitive to absolute intensity values?

Image gradients

Use pixel differences

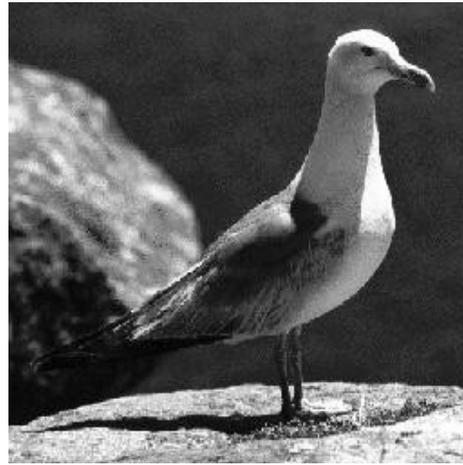
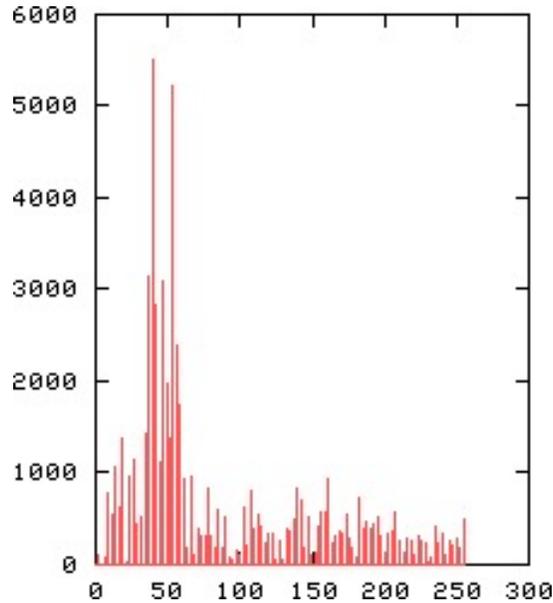


Feature is invariant to absolute intensity values

What are the problems?

How can you be less sensitive to deformations?

Image Representations: Histograms



Motivation: We want some sensitivity to spatial layout, but **not too much**, so blocks of histograms give us that

Global histogram to represent distribution of features

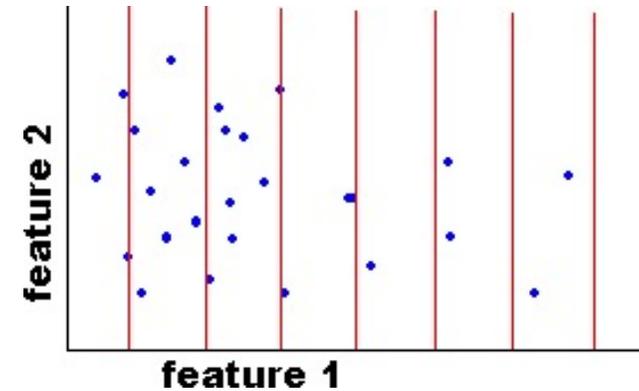
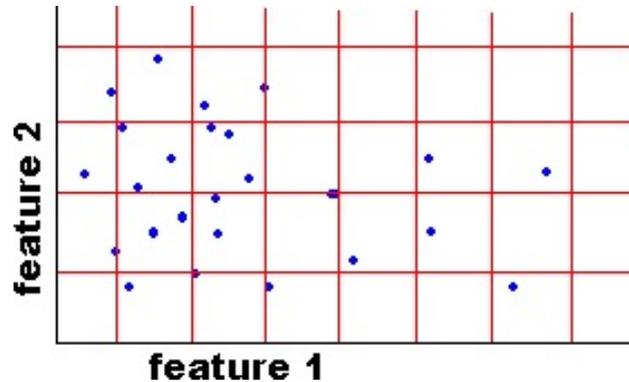
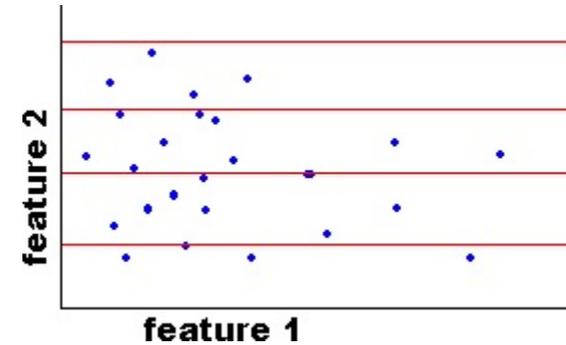
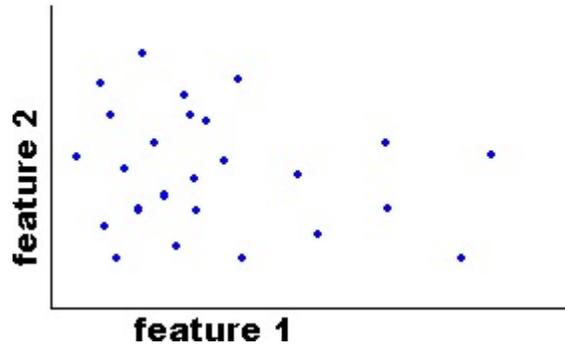
– How ‘well exposed’ a photo is

What about a local histogram per detected point?



Image Representations: Histograms

Histogram: Probability or count of data in each bin



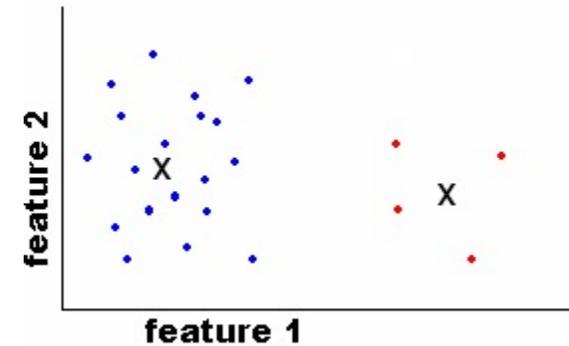
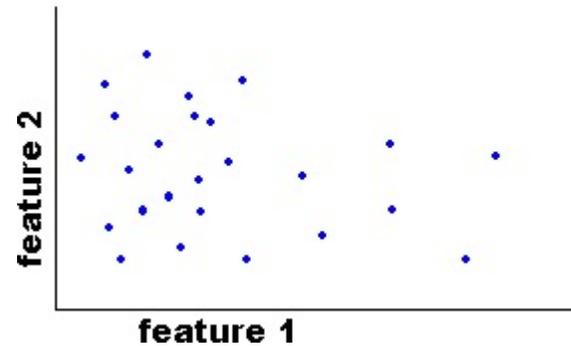
- Joint histogram
 - Requires lots of data
 - Loss of resolution to avoid empty bins

Marginal histogram

- Requires independent features
- More data/bin than joint histogram
- **The definition of feature can vary...**

Image Representations: Histograms

Clustering



Use the same cluster centers for all images

... so two different images' histograms can be "compared" (using what metric?)

Computing histogram distance

$$\text{histint}(h_i, h_j) = 1 - \sum_{m=1}^K \min(h_i(m), h_j(m))$$

Histogram intersection (assuming normalized histograms)

$$\chi^2(h_i, h_j) = \frac{1}{2} \sum_{m=1}^K \frac{[h_i(m) - h_j(m)]^2}{h_i(m) + h_j(m)}$$

Chi-squared Histogram matching distance



Cars found by color histogram matching using chi-squared

Histograms: Implementation issues

- Quantization
 - Grids: fast but applicable only with few dimensions
 - Clustering: slower but can quantize data in higher dimensions



Few Bins

Need less data

Coarser representation

Many Bins

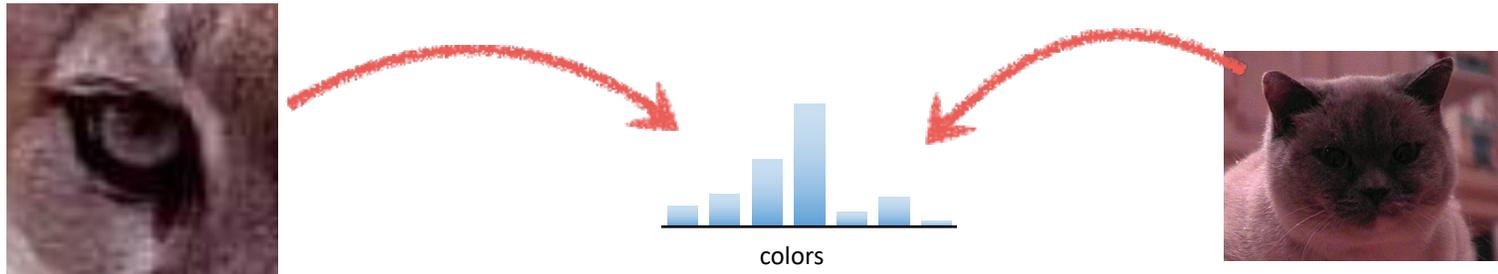
Need more data

Finer representation

- Matching
 - Histogram intersection or Euclidean may be faster
 - Chi-squared often works better
 - Earth mover's distance is good for when nearby bins represent similar values

Intensity/Color histogram

Count the colors in the image using a histogram



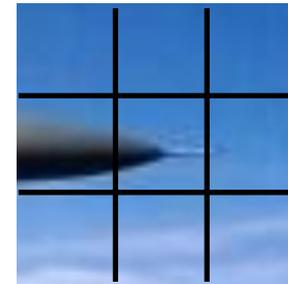
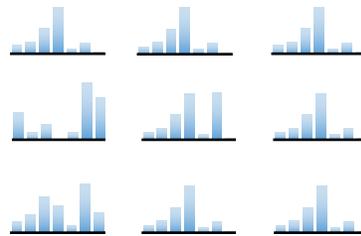
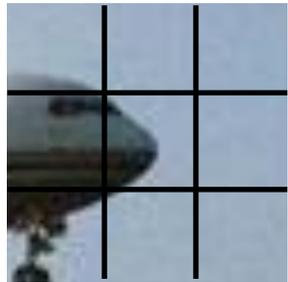
Invariant to changes in scale and rotation

What are the problems?

How can you be more sensitive to spatial layout?

Spatial histograms

Compute histograms over spatial 'cells'



Retains rough spatial layout

Some invariance to deformations

What are the problems?

How can you be completely invariant to rotation?

For what things might we compute histograms?

- *From Color to Texture/Keypoints*
- Histograms of oriented gradients (HOG)
- Scale Invariant Feature Transform (SIFT)
 - Extremely popular (63k citations in 2021)

IMHO, one of the most elegant designs ever in CV

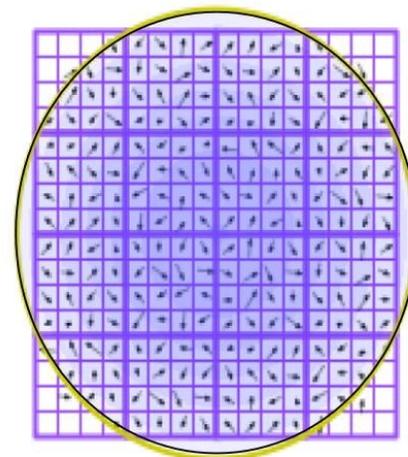
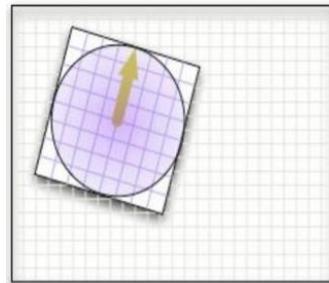
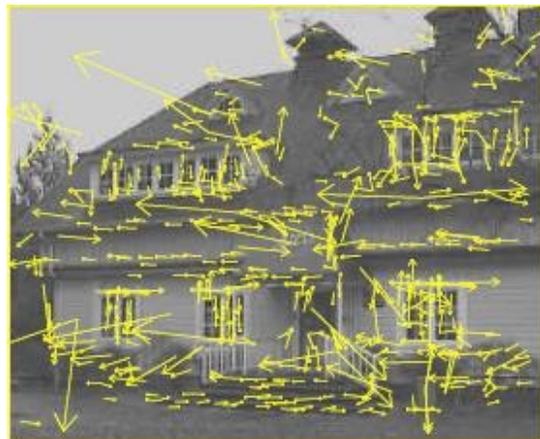
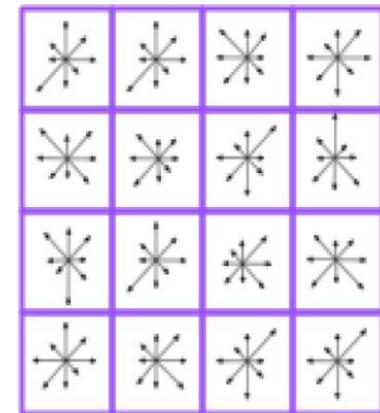


Image gradients



Keypoint descriptor

SIFT – Lowe IJCV 2004

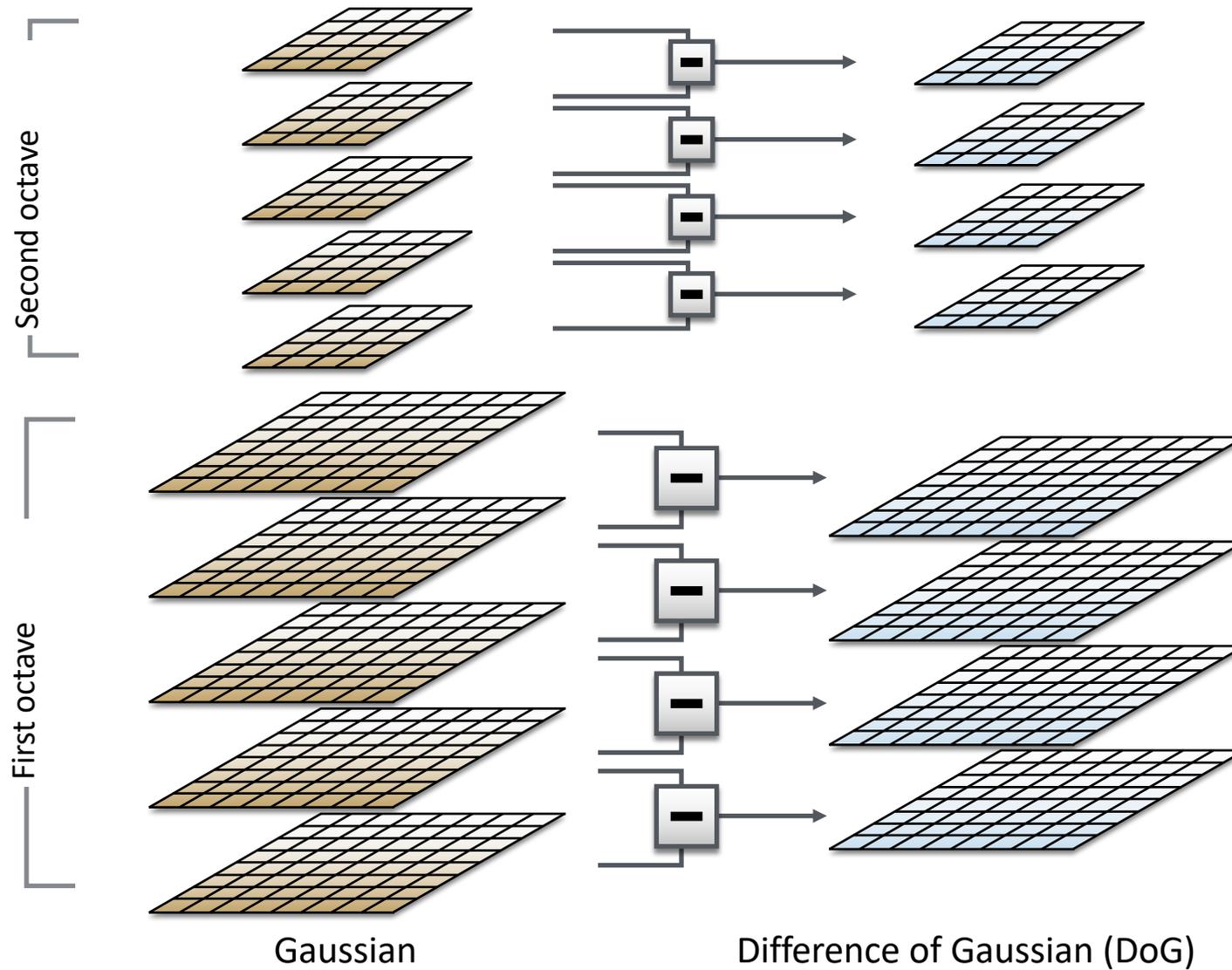


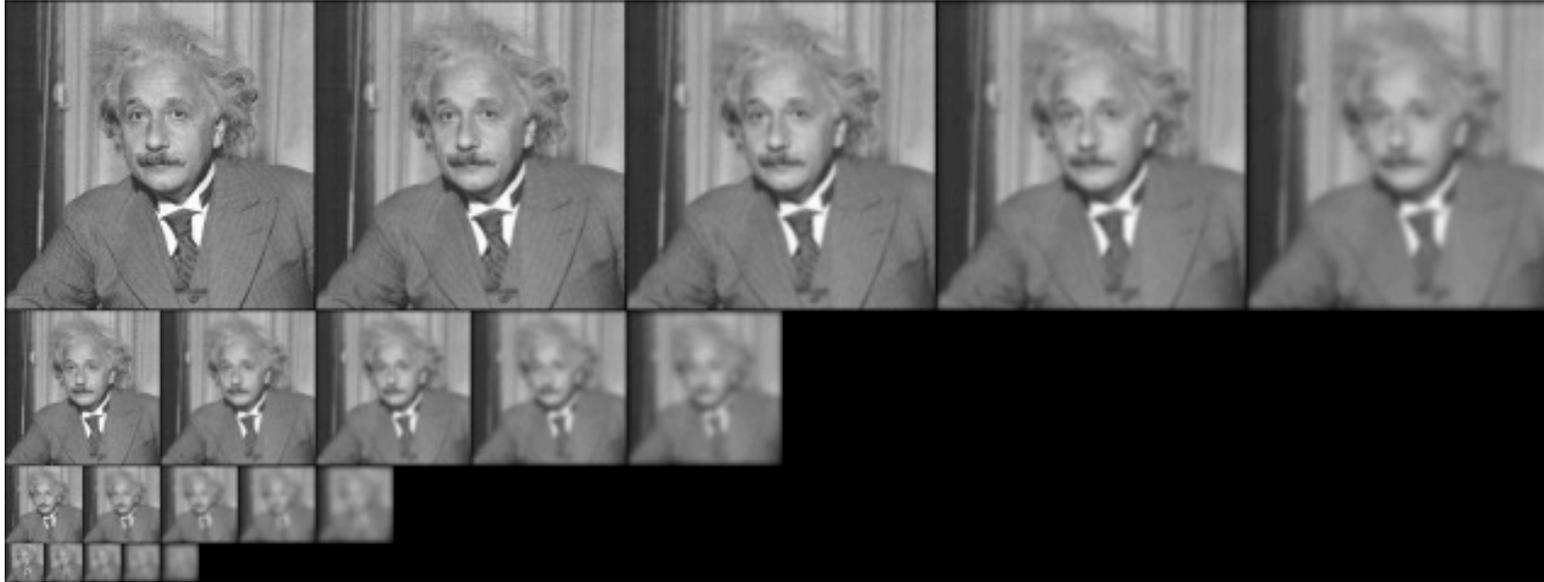
SIFT (Scale Invariant Feature Transform)

SIFT describes both a **detector** and **descriptor**

1. Multi-scale extrema detection
2. Keypoint localization
3. Orientation assignment
4. Keypoint descriptor

1. Multi-scale extrema detection



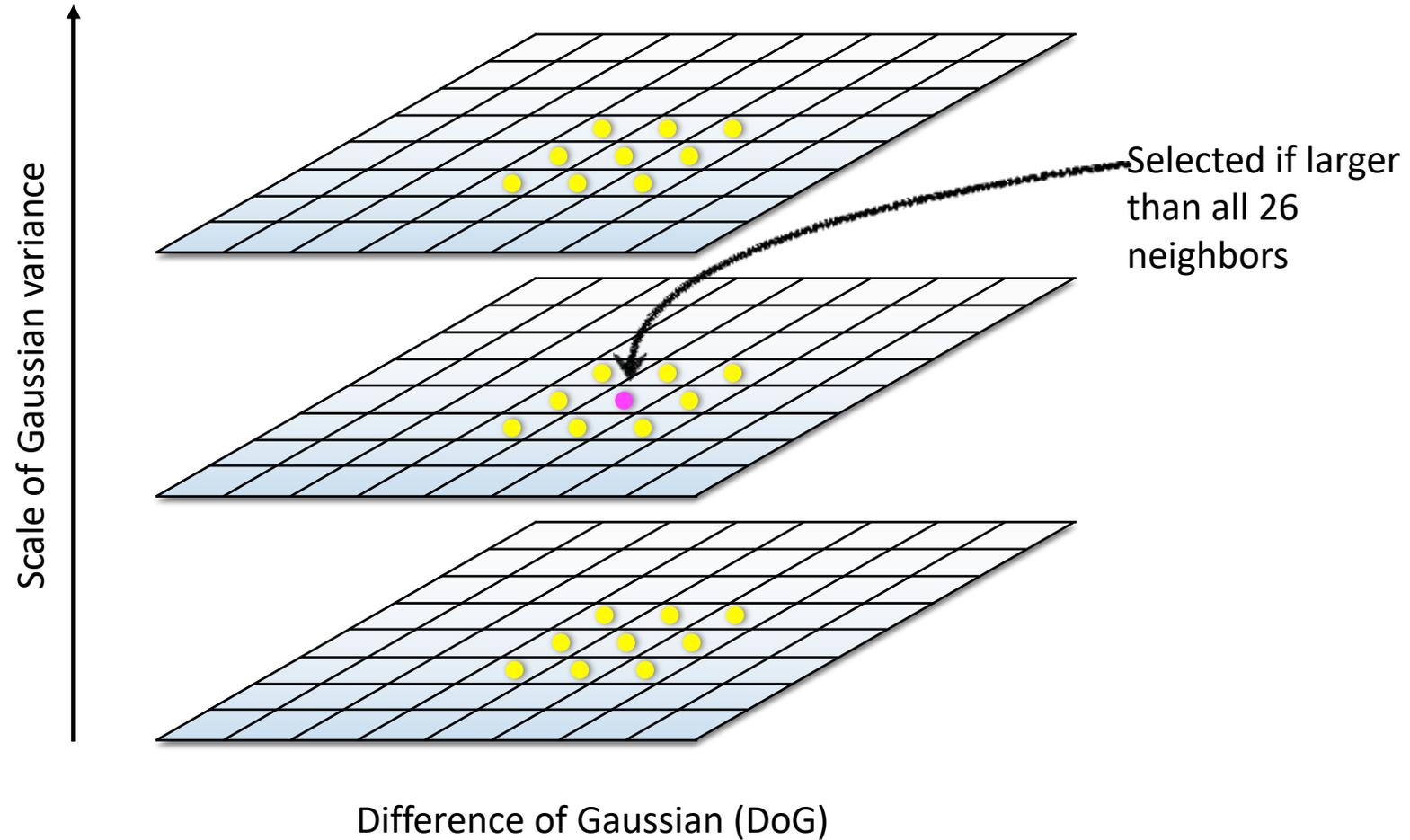


Gaussian



Laplacian

Scale-space extrema



2. Keypoint localization (why this step?)

- Reject flats:

- $|D(\hat{\mathbf{x}})| < 0.03$

- Reject edges:

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Let α be the eigenvalue with larger magnitude and β the smaller.

$$\text{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\text{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

Let $r = \alpha/\beta$.
So $\alpha = r\beta$

$$\frac{\text{Tr}(\mathbf{H})^2}{\text{Det}(\mathbf{H})} = \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(r\beta + \beta)^2}{r\beta^2} = \frac{(r + 1)^2}{r},$$

$(r+1)^2/r$ is at a min when the 2 eigenvalues are equal.

- $r < 10$

3. Orientation assignment

For a keypoint, **L** is the **Gaussian-smoothed** image at its selected scale,

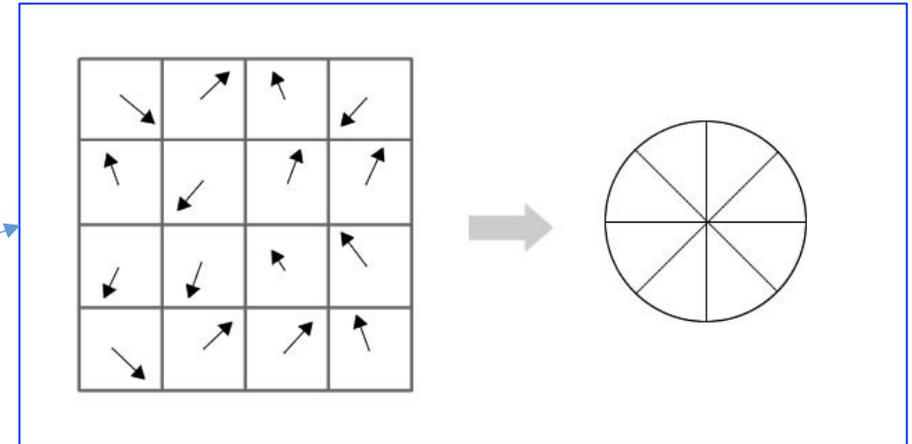
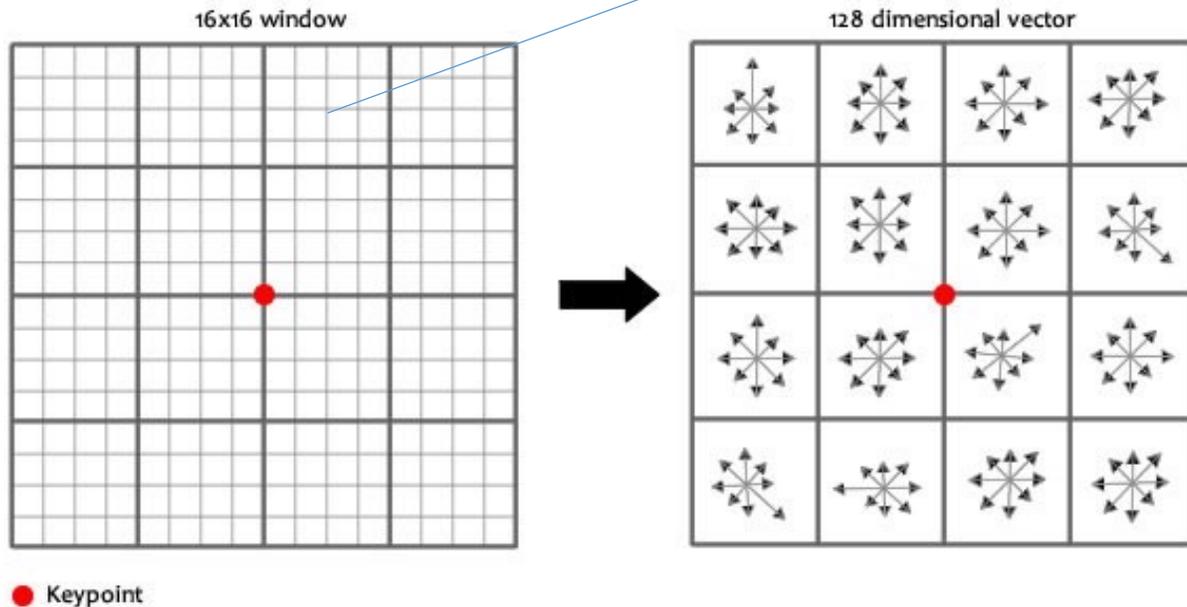
$$m(x, y) = \sqrt{\underbrace{(L(x + 1, y) - L(x - 1, y))^2}_{\text{x-derivative}} + \underbrace{(L(x, y + 1) - L(x, y - 1))^2}_{\text{y-derivative}}}$$
$$\theta(x, y) = \tan^{-1}((L(x, y + 1) - L(x, y - 1)) / (L(x + 1, y) - L(x - 1, y)))$$

Detection process returns $\{x, y, \sigma, \theta\}$
location scale orientation

4. Keypoint descriptor

At this moment, each keypoint has {location, scale, orientation}...

- Use local image gradients at selected scale and rotation to describe each keypoint region.



Adding more invariances to SIFT

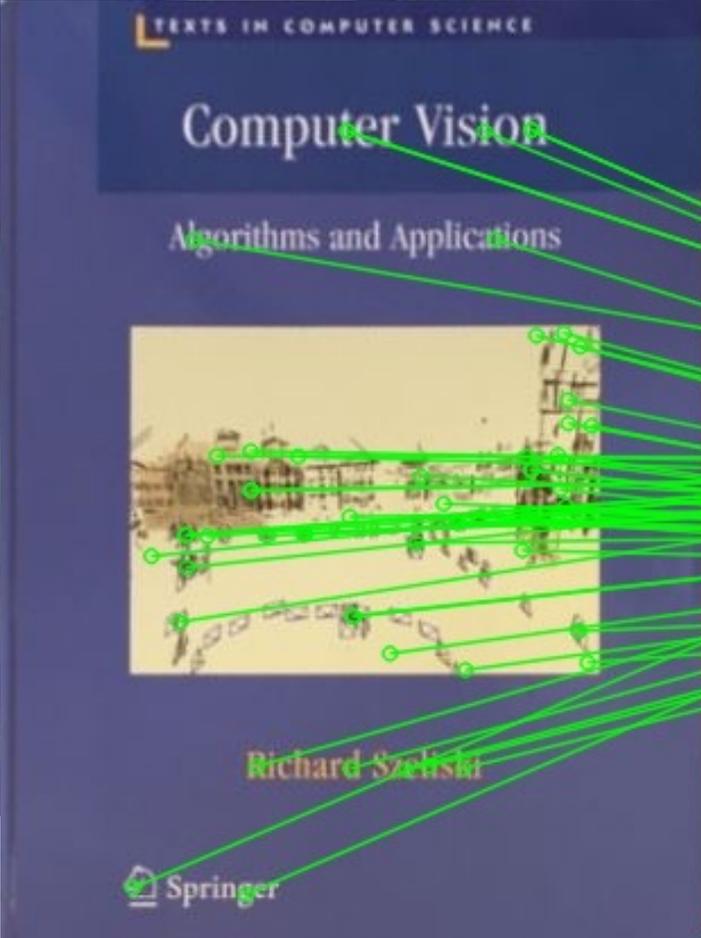
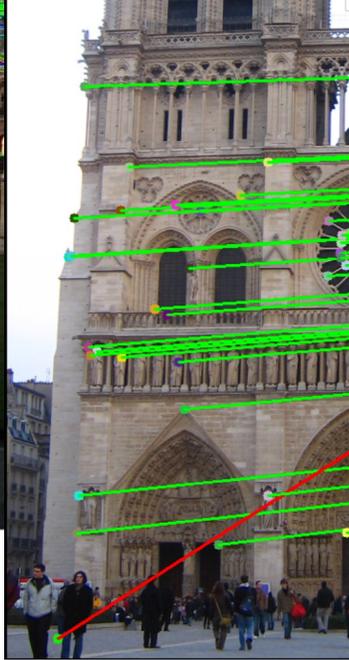
Rotation Invariance:

- The feature vector uses gradient orientations. So if you rotate the image, everything changes!
- SIFT adopts “**relative rotation**”: the keypoint’s own rotation is subtracted from each orientation. Thus each gradient orientation is relative to the keypoint’s orientation.

Illumination Invariance:

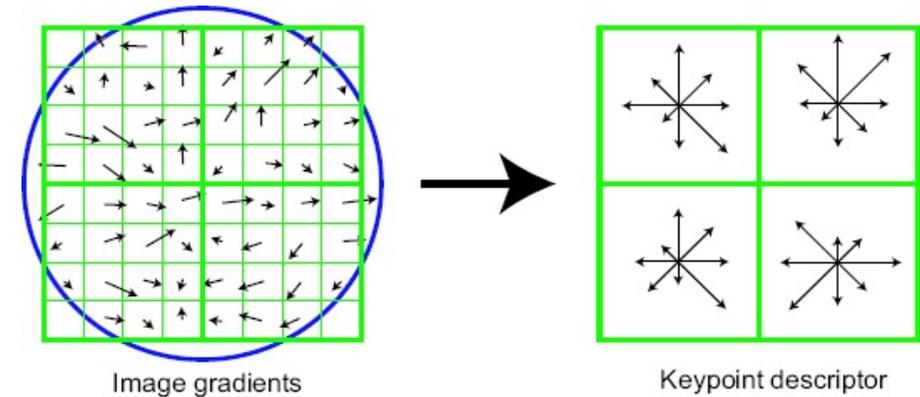
- All keypoints’ 128-dim vector are normalized to 1
- Sometimes we have “outlier illumination” ...
 - Practically, after normalization, we clamp all gradients > 0.2 , then renormalize to $[0,1]$

*SIFT achieves an extremely elegant and robust balance between **global layout** (histogram) versus **local feature** (full gradient), discriminativeness versus resilience*



Review: Local Descriptors

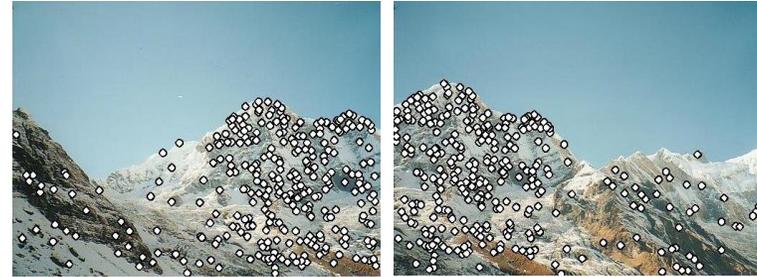
- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
 - Robust and Distinctive
 - Compact and Efficient
- Most available descriptors focus on edge/gradient information
 - Capture texture information
 - Color rarely used



Local features: main components

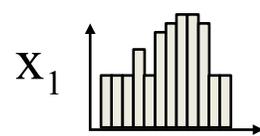
1) Detection:

Find a set of distinctive key points.

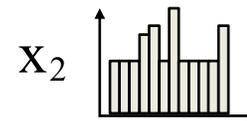
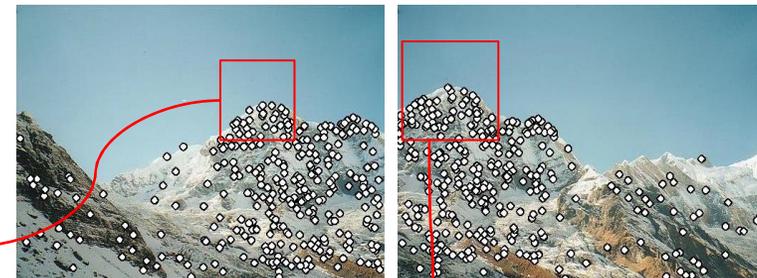


2) Description:

Extract feature descriptor around each interest point as vector.



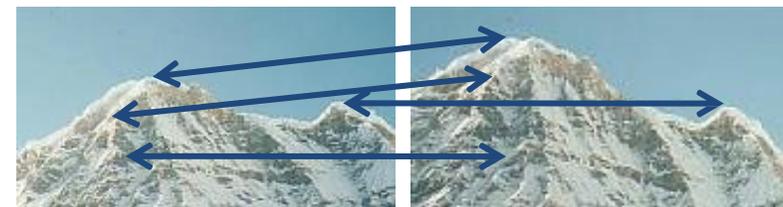
$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

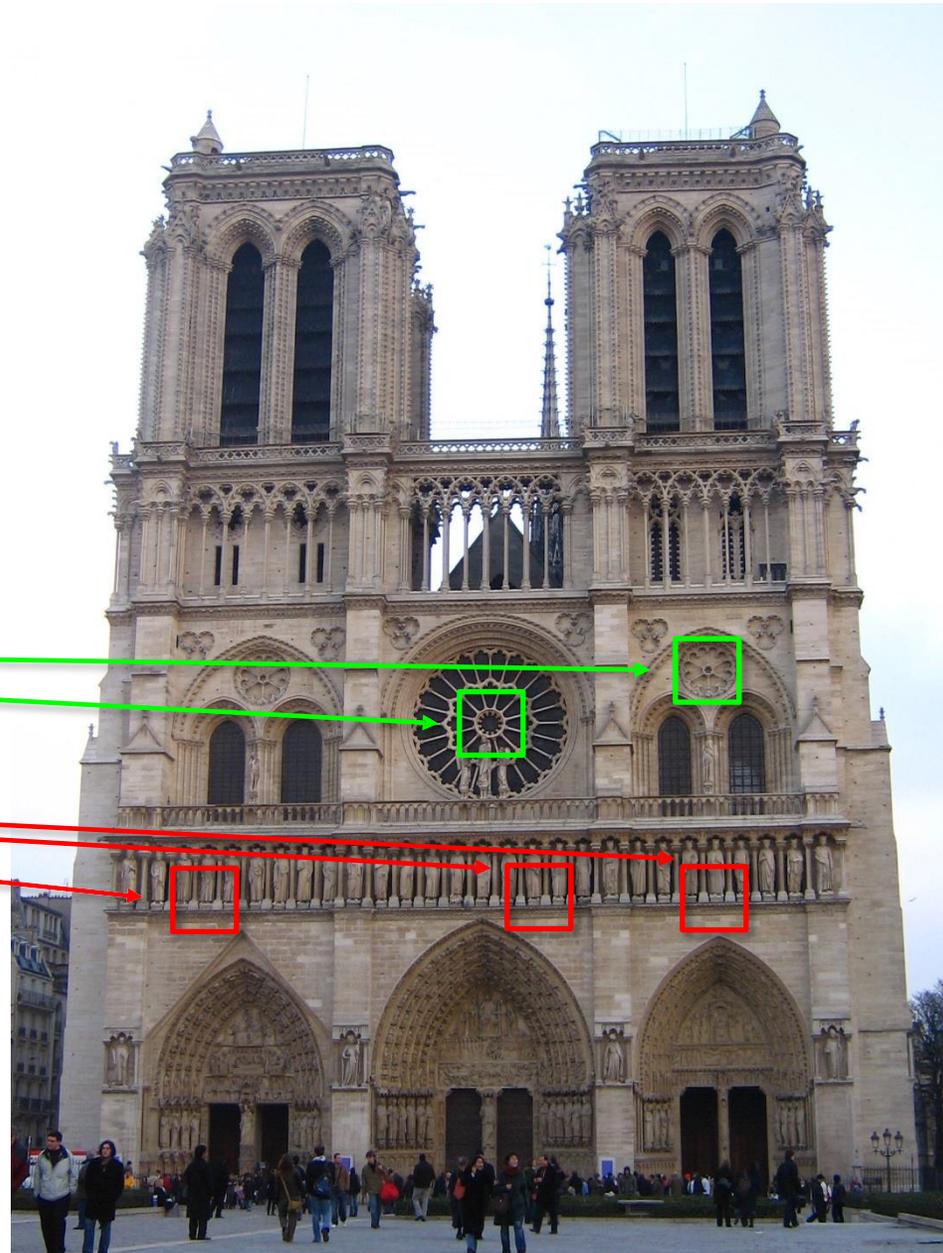
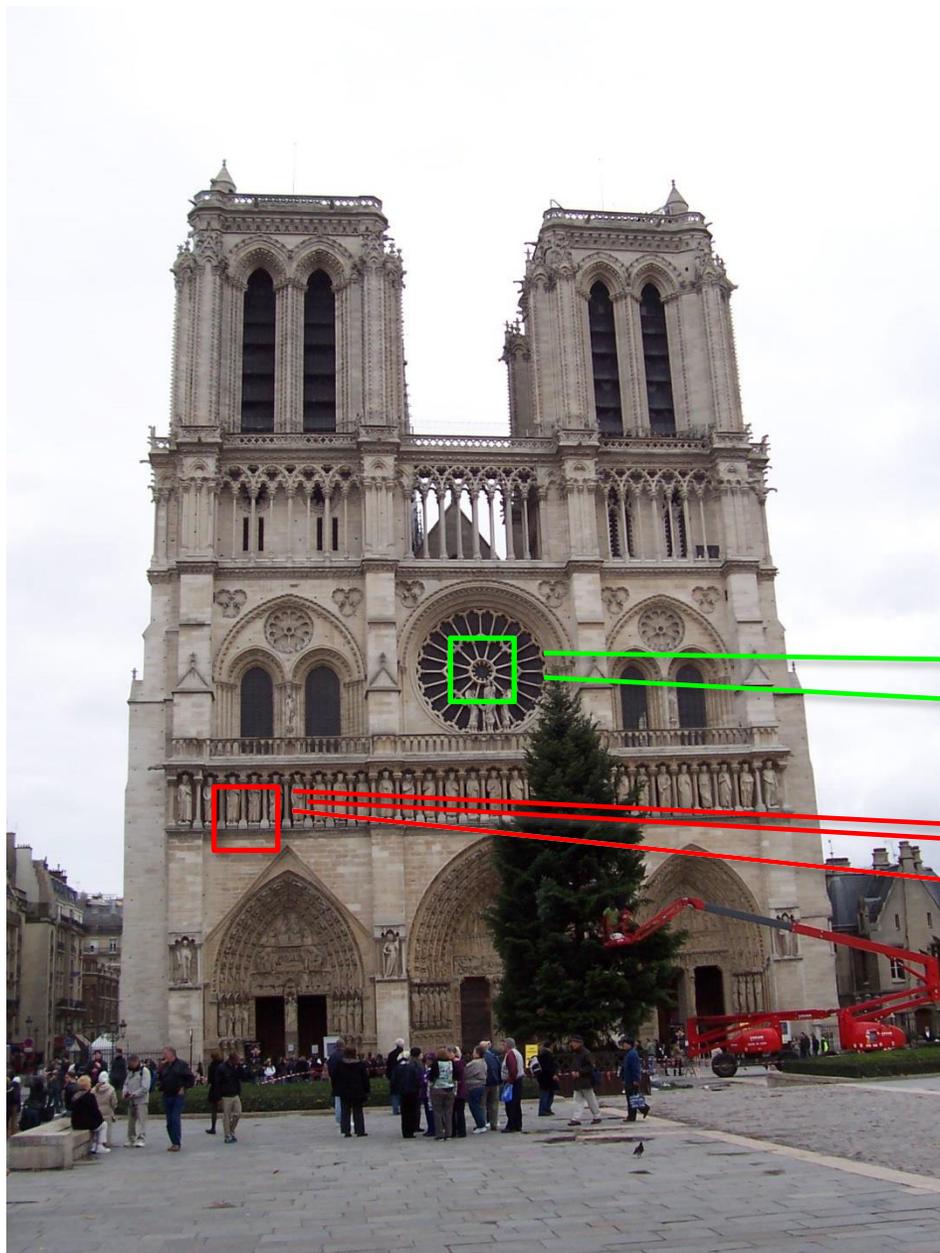
3) Matching:

Compute distance between feature vectors to find correspondence.



Ok, now we have local features...

But how similar can the two features be called "match"?

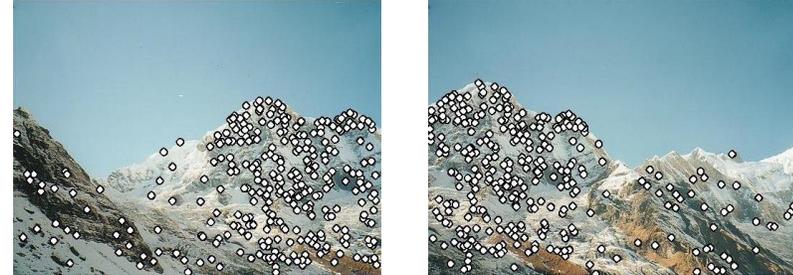


Distance: 0.34, 0.30, 0.40

Distance: 0.61, 1.22

What to Consider in the Design of Feature Matching

- Two images, I_1 and I_2



- Two sets X_1 and X_2 of feature points
 - Each feature point \mathbf{x}_1 has a descriptor $\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$
- Distance, bijective/injective/surjective, noise, confidence, computational complexity, generality...

Euclidean distance vs. Cosine Similarity

- Euclidean distance:

$$d(\mathbf{p}, \mathbf{q}) = d(\mathbf{q}, \mathbf{p}) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \cdots + (q_n - p_n)^2}$$

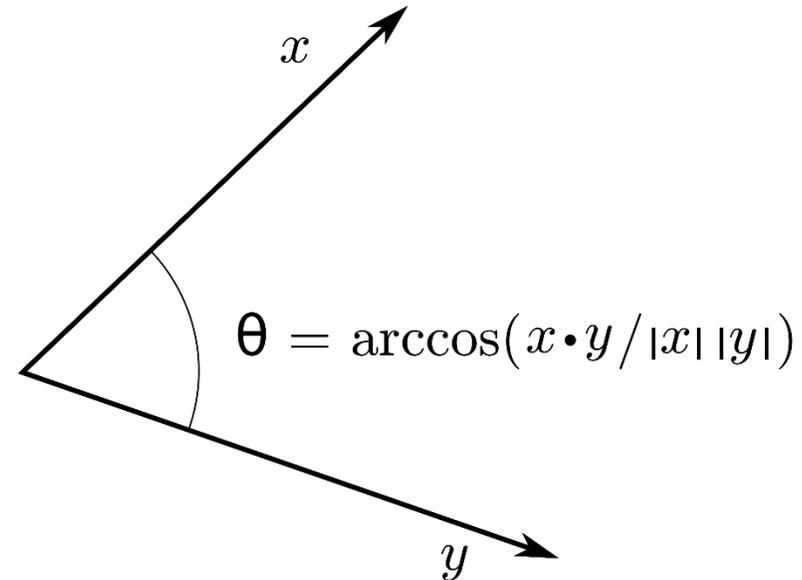
$$= \sqrt{\sum_{i=1}^n (q_i - p_i)^2}.$$

$$\|\mathbf{q} - \mathbf{p}\| = \sqrt{(\mathbf{q} - \mathbf{p}) \cdot (\mathbf{q} - \mathbf{p})}.$$

- Cosine similarity:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|_2 \|\mathbf{b}\|_2 \cos \theta$$

$$\text{similarity} = \cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\|_2 \|\mathbf{B}\|_2}$$



Feature Matching

- Criteria 1:
 - Compute distance in feature space, e.g., Euclidean distance between 128-dim SIFT descriptors
 - Match point to lowest distance (nearest neighbor)
- Problems:
 - Does everything have a match?

Feature Matching

- Criteria 2:
 - Compute distance in feature space, e.g., Euclidean distance between 128-dim SIFT descriptors
 - Match point to lowest distance (nearest neighbor)
 - Ignore anything higher than threshold (no match!)

- Problems:
 - Threshold is hard to pick
 - Non-distinctive features could have lots of close matches, only one of which is correct

Nearest Neighbor Distance Ratio

Compare distance of closest (NN1) and second-closest (NN2) feature vector neighbor.

- If $NN1 \approx NN2$, ratio $\frac{NN1}{NN2}$ will be ≈ 1 -> matches too close.
- As $NN1 \ll NN2$, ratio $\frac{NN1}{NN2}$ tends to 0.

Sorting by this ratio puts matches in order of confidence.

Threshold ratio – but how to choose?

- depends on your application's view on the trade-off between the number of false positives and true positives! [You need to tune...](#)

Visual Similarity is still/forever an OPEN problem



"While they're similar,
same and similar don't
mean the same thing."

-Robert Lee Brewer





The University of Texas at Austin
**Electrical and Computer
Engineering**
Cockrell School of Engineering